King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101
Final Exam
Term 153
Wednesday, August 31, 2016

EXAM COVER

Number of versions: 4
Number of questions: 28
Number of Answers: 5 per question

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Math 101
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Term 153
Wednesday, August 31, 2016
Net Time Allowed: 180 minutes

MASTER VERSION

1. The value of the limit $\lim_{x \to 2^+} \frac{1-x}{2-x}$ is equal to:

(a) $+\infty$

(b) $-\infty$

(c) $+1$

(d) -1

(e) 0

2. If $|x-2| \leq g(x) \leq 3\sec^2(\pi x) - 2$, then $\lim_{x \to 1} g(x)$ is equal to:

(a) 1

(b) 0

(c) -1

(d) Does not exist

(e) ∞

3. The largest positive number δ such that $|2x-4| < 0.1$ whenever $0 < |x-2| < \delta$ is equal to

(a) 0.05

(b) 0.1

(c) 0.5

(d) 0.01

(e) 0.005

4. If $f(x) = \begin{cases} a - \dfrac{\sin x}{x} & -1 \le x < 0 \\ x^2 - \sqrt{x} + b & 0 \le x < 4 \\ 5(\dfrac{x^2-1}{x+1}) & 4 \le x \le 5 \end{cases}$

satisfies the hypotheses of the Intermediate Value Theorem, then $b^a =$

(a) 1

(b) 0

(c) 3

(d) 5

(e) 4

5. If f is differentiable, then $\lim_{h\to 0} \dfrac{f(2x-h)-f(2x-3h)}{h} =$

(a) $2\,f'(2x)$

(b) $-2\,f'(x)$

(c) $-2\,f'(2x)$

(d) $f'(x)$

(e) $-f'(x)$

6. If the average rate of change of $y = \dfrac{1}{\sqrt{4-5x}}$ with respect to x on the interval $[-1,0]$ is equal to c, then $c =$

(a) $\dfrac{1}{6}$

(b) $-\dfrac{29}{36}$

(c) $-\dfrac{1}{3}$

(d) $\dfrac{31}{36}$

(e) $-\dfrac{2}{3}$

7. The function $f(x) = \dfrac{\sqrt[3]{x}}{1-x^2}$ has a vertical tangent at $x =$

(a) 0

(b) 1

(c) -1

(d) ∓ 1

(e) 4

8. If the position function of a particle is given by
$s(t) = 2t^3 - 9t^2 + 12t + 1$, then the distance travelled by the particle during the interval time $[1, 3]$ is:

(a) 6

(b) 4

(c) 5

(d) 10

(e) 1

9. If $y = \sqrt{x}(x - 3)$, then y is

(a) decreasing on $(0, 1)$ and increasing on $(1, +\infty)$.

(b) increasing on $(0, 1)$ and decreasing on $(1, +\infty)$.

(c) increasing on $(0, 1)$ and increasing on $(1, +\infty)$.

(d) decreasing on $(0, 1)$ and decreasing on $(1, +\infty)$.

(e) increasing on $(0, +\infty)$.

10. The graph of $y = \sec x + \csc x, 0 < x < \frac{\pi}{2}$, has a horizontal tangent at $x =$

(a) $\dfrac{\pi}{4}$ only

(b) $\dfrac{\pi}{3}$ and $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{3}$ only

(d) $\dfrac{\pi}{6}$ and $\dfrac{\pi}{4}$

(e) $\dfrac{\pi}{6}$ only

11. The slope of the tangent line to the graph of $y = \ln(1 + x + x^2)^3$ at $x = 1$ is equal to

(a) 3

(b) $\frac{1}{3}$

(c) 1

(d) $\ln 3$

(e) 0

12. If $f(x) = e^{x+\sin x}$, then $f'(\pi) =$

(a) 0

(b) $e^{-\pi}$

(c) e^{π}

(d) 1

(e) -1

13. The slope of the tangent line to the curve $y^y = x^x$ at the point (e, e) is

(a) 1

(b) 0

(c) -1

(d) e

(e) e^{-1}

14. If $f(x) = \sqrt{1 + 2^{x+x^2}}$ then $f'(0) =$

(a) $\dfrac{\ln 2}{2\sqrt{2}}$

(b) $\dfrac{\ln 2}{2}$

(c) $\dfrac{\ln 2}{\sqrt{2}}$

(d) $\dfrac{1}{2\sqrt{2 \ln 2}}$

(e) $\dfrac{1}{2 \ln 2}$

15. If $f(x) = x^{100} + 3\sin x$ then $f^{(358)}\left(\frac{\pi}{6}\right) =$

(a) $\frac{-3}{2}$

(b) $\frac{1}{2}$

(c) $\frac{3\sqrt{2}}{2}$

(d) 0

(e) $\frac{3\sqrt{3}}{2}$

16. If the sides of a rectangle are increasing at the same rate of $\frac{1}{4}\, m/s$, then how fast is the area of the rectangle increasing when the sides are 4 and 8.

(a) 3

(b) 2

(c) 1

(d) 4

(e) 8

17. The radius of a cone was measured and found to be $3cm$ with a possible **relative** error of $\frac{0.03}{3}$. If the height of the cone is measured to be triple of the radius, then the **relative** error of the volume of the cone is:

$\left[\text{Hint: } V = \frac{1}{3}\pi r^2 h\right]$

(a) 0.03

(b) 0.01

(c) $(0.03)\,\pi$

(d) $(0.01)\,\pi$

(e) $3\,\pi$

18. The curve $y = \cosh(\ln x) + 4x$ has a horizontal tangent line at $x =$

(a) $\frac{1}{3}$

(b) $-\frac{1}{3}$

(c) $\frac{1}{6}$

(d) 3

(e) -3

19. If $f(x) = \tan^{-1}(\sinh x)$ then $f'(x)$ is

(a) $\operatorname{sech} x$

(b) $\operatorname{csch} x$

(c) $\tanh x$

(d) $\coth x$

(e) $\cosh x$

20. If a and b represent the absolute maximum and the absolute minimum of the function $f(x) = \frac{x^3}{3} + \frac{x^2}{2} - 2x + 1$ on the interval $[0, 2]$, then $a + b =$

(a) $\frac{3}{2}$

(b) $\frac{5}{3}$

(c) $\frac{2}{3}$

(d) $\frac{8}{3}$

(e) $\frac{5}{2}$

21. If $f(x) = 4 + \sqrt{x-1}$, then the value of c guaranteed by the mean value theorem on $[1, 5]$ is

(a) 2

(b) 1

(c) 0

(d) 6

(e) 4

22. In the interval $[-2, 2]$, the equation $x^3 - 15x + 20 = 0$ has

(a) exactly one root

(b) at least one root

(c) at most two roots

(d) at least two roots

(e) no root

23. Let $y = f(x) = ax^3 + bx^2 - 9x + c$, where a, b and c are constants. If f has local maximum at $x = -1$, an inflection point at $x = 1$, and $y-$ intercept equals to 1, then

(a) $a = 1\,,\, b = -3\,, and\, c = 1$

(b) $a = 1\,,\, b = 0\,, and\, c = 1$

(c) $a = 1\,,\, b = -3\,, and\, c = 0$

(d) $a = -1\,,\, b = 0\,, and\, c = 1$

(e) $a = -1\,,\, b = 3\,, and\, c = 1$

24. $\lim_{x \to 0} \left[\frac{1}{x(x+1)} - \frac{\ln(1+x)}{x^2} \right] =$

(a) $-\frac{1}{2}$

(b) 0

(c) 1

(d) ∞

(e) $\frac{1}{2}$

25. The graph of the function $f(x) = 2\ \ln(1 + x^2) + 3\ \tan^{-1}\ x$ is

(a) concave up on $\left(-2, \dfrac{1}{2}\right)$ concave down on $(-\infty, -2)$ & $\left(\dfrac{1}{2}, \infty\right)$

(b) concave down on $\left(-2, \dfrac{1}{2}\right)$ concave up on $(-\infty, -2)$ & $\left(\dfrac{1}{2}, \infty\right)$

(c) concave up on $(-\infty, -2)$ concave down on $\left(\dfrac{1}{2}, \infty\right)$

(d) concave down on $(-\infty, -2)$ concave up on $\left(\dfrac{1}{2}, \infty\right)$

(e) always concave down

26. The height of a right circular cone is $4cm$ and its radius is $2cm$. The dimensions of the right circular cylinder with the maximum volume that can be inscribed in the cone is:

(a) radius $= \dfrac{4}{3}$ height $= \dfrac{4}{3}$

(b) radius $= \dfrac{2}{3}$ height $= \dfrac{2}{3}$

(c) radius $= \dfrac{2}{3}$ height $= \dfrac{4}{3}$

(d) radius $= \dfrac{4}{9}$ height $= \dfrac{4}{9}$

(e) radius $= \dfrac{4}{3}$ height $= \dfrac{4}{9}$

27. If we use Newton's method to find an approximate solution for $x - 2\cos x = 0$ starting with $x_1 = \dfrac{\pi}{2}$, then the next approximate solution is $x_2 =$

(a) $\dfrac{\pi}{3}$

(b) 0

(c) π

(d) $\dfrac{\pi}{4}$

(e) $\dfrac{\pi}{6}$

28. If $f'(x) = \dfrac{(1+3\sqrt{x})^2}{x}$ then the most general antiderivative is

(a) $\ln|x| + 12\sqrt{x} + 9x + C$

(b) $\ln|x| + 12\sqrt{x} + 9x^2 + C$

(c) $\ln|x| + 6\sqrt{x} + 9x + C$

(d) $\ln|x| + 3\sqrt{x} + 9x + C$

(e) $\ln|x| + 6\sqrt{x} + 9x^2 + C$

Q	MM	V1	V2	V3	V4
1	a	d	e	b	e
2	a	c	c	b	c
3	a	d	c	b	c
4	a	c	e	a	b
5	a	c	b	e	e
6	a	d	e	c	d
7	a	c	e	d	e
8	a	d	c	b	c
9	a	a	b	a	e
10	a	e	d	c	d
11	a	b	d	d	e
12	a	a	e	b	b
13	a	c	c	e	b
14	a	a	c	e	d
15	a	a	c	a	b
16	a	e	a	b	a
17	a	d	c	b	b
18	a	a	e	d	d
19	a	c	b	e	d
20	a	e	a	a	b
21	a	d	c	d	e
22	a	e	d	e	a
23	a	e	e	e	b
24	a	a	c	a	d
25	a	c	a	e	a
26	a	a	b	e	e
27	a	e	c	e	c
28	a	a	b	c	d