King Fahd University Of Petroleum & Minerals
Department Of Mathematics And Statistics
STAT502 :Statistical Inference (152)

Final Exam **Thursday May 19, 2016**

Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	25	
Two	8	
Three	10	
Four	10	
Five	12	
Six	15	
Seven	18	
Eight	14	
Nine	20	
Ten	15	
Eleven	20	
Twelve	18	
Total	185	

Question.1 (**6+5+6+8=25-Points**)

Let $X_1, X_2, \ldots, X_n$ be *iid* $N(\mu, 1)$, and let $d(\mu) = \mu^2$

(a) Shout that $\bar{X}$ is a complete sufficient steatitic for μ.

(b) Show that the minimum variance of any estimator of μ^2 from the FCR inequality is $\frac{4\mu^2}{n}$.

(c) Show that $T(\boldsymbol{X}) = \bar{X}^2 - \dfrac{1}{n}$ is the UMVUE of μ^2

(d) Verify that the variance of $T(\boldsymbol{X})$ is $\frac{4\mu^2}{n} + \frac{2}{n^2}$

Question.2 (**8-Points**)

Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with PDF given by: $\begin{cases} \theta(1-x)^{\theta-1} & ,0 < x < 1 \\ 0, & \text{Otherwise} \end{cases}$,

and $\theta \geq 1$. Find the MLE of θ.

Question.3 (**10-Points**)

Let $\pi(\theta)$ be the prior distribution of θ, $h(\theta|\boldsymbol{x})$ be the posterior distribution of θ given $\boldsymbol{X} = \boldsymbol{x}$. Assume that $f(\boldsymbol{x};\theta) = g(\boldsymbol{x}).h(\theta|\boldsymbol{x})$ Let $L(\theta, a) = w(\theta)(t(\theta) - a)^2$, $w(\theta) > 0$ for all θ. Prove that the Bayes' estimator of $t(\theta)$ is given by $\delta^*(\boldsymbol{x}) = \dfrac{E\left(w(\theta)t(\theta)|\boldsymbol{x})\right)}{E\left(w(\theta)|\boldsymbol{x}\right)}$.

Question.4 (**4+2+4=10-Points**)

Let $X_1, X_2, \ldots, X_n$ be *iid* $b(1, \theta)$. Assume the prior distribution of the θ over $\Theta = (0, 1)$ is $\beta(\alpha, \beta)$.

(a) Find the posterior distribution $h(\theta|\boldsymbol{x})$.

(b) Find the Bayes' estimator of θ assuming the squared error loss function is used.

(c) Find the Mean Squared Error (MSE) of the Bayes' estimator.(**Do not simplify**)

Question.5 (**7+5=12-Points**)

Let X bea single observation from the PDF given by $f(x,\beta) = \begin{cases} \beta x^{\beta-1}, & 0 < x < 1 \\ 0, & \text{Otherwise} \end{cases}$

(a) Find the Neyman-Pearson MP size α test of $H_0 : \beta = 1$ aganist $H_1 : \beta = \beta_1 (\beta_1 > 1)$.

(b) Find the probability of **Type I** and **Type II** errors.

Question .6 **(8+4+3=15-Points)**

Let X be a random variable with PMF under H_0 and H_1 is given by:

x	1	2	3	4	5	6	7
$f_0(x)$	0.01	0.01	0.01	.01	.01	0.01	0.94
$f_1(x)$	0.06	0.05	0.04	0.03	0.02	0.01	0.79

(a) Find the MP size $\alpha = 0.04$ test for H_0 against H_1.

(b) Find the probability of Type II error.

(c) Find the power of the test.

Question .7 **(6+8+4=18-Points)**

Let X have the logistic distribution with PDF $f_\theta(x) = \dfrac{e^{x-\theta}}{(1+e^{x-\theta})^2}$, $-\infty < x, \theta < \infty$ and a CDF given by $F_\theta(x) = \dfrac{e^{x-\theta}}{1+e^{x-\theta}}$

(a) Show that the family of distribution given by x has an MLR.

(b) Find the UMP size α test for testing $H_0 : \theta \leq 0$ paganist $H_1 : \theta > 0$

(c) If $\alpha = 0.20$ and $x = 1.25$, what is your dicsion?Why?

Question .8 **(14-Points)**

Let X be one observation from the PDF $f_\theta(x) = \dfrac{2}{\theta^2}(\theta - x), 0 < x < \theta$ and 0 elsewhere, where $\theta > 0$. Find the GLR test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

Question .9 **(20-Points)**

Back to **Question.8**, invert the test to obtain an equal-tailed $(1-\alpha)100\%$ confidence interval for θ.

Question .10 **(7+8=15-Points)**

A sample of 15 observations has a mean of 57.6 and a variance of 1.8. A further sample of 10 values has a mean of 55.4 and a variance of 2.05. Assume the two populations are normal.

(a) Test the equality of the two populations variances at $\alpha = 0.05$.

(b) Test the hypothesis that the two samples come from the same normal distribution at $\alpha = 0.05$.

Question .11 **(20-Points)**

Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with PDF $f_\theta(x) = \begin{cases} \frac{2}{\theta} x e^{-x^2/\theta}, & x > 0, \theta > 0 \\ 0, & \text{Otherwise} \end{cases}$

(a) Verify that $Y = \frac{X^2}{\theta}$ is a pivotal quantity. (Hint: Find the distribution of Y)

(b) Use the pivot $T(\boldsymbol{x}, \theta) = \sum_{i=1}^{n} \frac{X_i^2}{\theta}$ to find a confidence set for θ, then obtain an equal-tailed $(1-\alpha)100\%$ confidence interval for θ

Question .12 **(18-Points)**

Let $X_1, X_2, \ldots, X_n$ be *iid* Poisson(λ). Assuming that λ is a value assumed by $G(\alpha, \beta)$ RV. Find an equal-tailed Bayesian confidence interval for λ.