King Fahd University Of Petroleum & Minerals
Department Of Mathematics And Statistics
STAT502 : Statistical Inference (152)
Second Exam **Sunday April 24, 2016**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	15	
Two	15	
Three	16	
Four	18	
Five	18	
Six	14	
Seven	8	
Eight	16	
Total	120	

Question.1 (**5+5+5=15-Points**)

Let $X_1, X_2, \ldots, X_n$ be *iid* from a population with finite mean μ and finite variance σ^2. Suppose that σ^2 is known. Assume that n is sufficiently large so that the central limit theorem holds. For testing the hypothesis $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1 (\mu_1 > \mu_0)$ consider the test $\phi(\boldsymbol{x}) = \begin{cases} 1, & \text{if } \bar{x} > k \\ 0, & \text{if } \bar{x} \leq k \end{cases}$

(a) Find k such that the test has size α approximately.

(b) Find probability of Type II error.

(c) Find the power of the test at $\mu = \mu_1$

Question.2 (**15-Points**)

A sample of size one is taken from the PDF $f_\theta(x) = \begin{cases} \frac{2}{\theta^2}(\theta - x), & 0 < x < \theta \\ 0, & \text{O.W} \end{cases}$. Find a MP test for testing $H_0 : \theta = \theta_0$ aganist $H_1 : \theta = \theta_1 (\theta_1 > \theta_0)$

Question.3 (**6+10=16-Points**)

Let $X_1, X_2, \ldots, X_n$ be *iid* from a PDF $f_\theta(x) = \theta x^{\theta-1}, \quad 0 < x < 1, \quad \theta > 0$ and 0 elsewhere.

(a) Show that the family has an MLR. (**Do not use the idea of exponential family**)

(b) Find a UMP size α test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$ and find the value of the constant that appears in your test.

Question.4 (**18-Points**)

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the PDF given by: $f_\theta(x) = \frac{1}{\theta^c} c x^{c-1} e^{-(\frac{x}{\theta})^c}$ $x > 0$ $c > 0$ is known. Find a UMP size α test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$, and find the value of the constant that appears in your test.

Question.5 (**18-Points**)

Let $X_1, X_2, \ldots, X_m$ be a random sample from $G(1, \beta)$. Find the GLR test for testing $H_0 : \beta = \beta_0$ against $H_1 : \beta \neq \beta_0$

Question .6 **(14-Points)**

Let $X_1, X_2, \ldots, X_n$ be a random sample from with PDF $f_\theta(x) = \dfrac{\theta}{x^{\theta+1}}$, $x > 1$, $\theta > 0$. Find the GLR for testing $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$. (**Hint**: You may assume $T = \ln(\Pi X_i)$, and hence $\Pi X_i = e^T$)

Question .7 **(8-Points)**

Three coins are tossed independently 200 times each with the following results:

Face	Coin 1	Coin 2	Coin 3
Heads	115	97	80
Tails	85	103	120

Do you think that the three coins are equally loaded? Test using $\alpha = 0.05$.

Question .8 **(8+8=16-Points)**

Two catalysts may be used in a batch chemical process. Twelve batches were prepared using catalyst 1, resulting in an average yield of 86 and a sample standard deviation of 3. Fifteen batches were prepared using catalyst 2, and they resulted in an average yield of 89 with a standard deviation of 2. Assume that yield measurements are approximately normally distributed.

(a) Test the equality of the two populations variances at level $\alpha = 0.05$ and write your conclusion.

(b) Is there evidence to support a claim that catalyst 2 produces a higher mean yield than catalyst 1? Use $\alpha = 0.01$.