King Fahd University Of Petroleum & Minerals
Department Of Mathematics And Statistics
STAT502 : Statistical Inference (152)
First Exam **Tuesday March 1, 2016**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	20	
Two	15	
Three	12	
Four	22	
Five	8	
Six	8	
Seven	15	
Total	100	

Question.1 (**16+4=20-Points**)

Let $X_1, X_2, \ldots, X_n$ be *iid* from PDF $f(x) = \begin{cases} e^{-(x-\mu)} & , -\infty < \mu < x < \infty \\ 0 & , \text{Otherwise} \end{cases}$

(a) Show that $T(\boldsymbol{x}) = X_{(1)} = \min_i\{X_1, X_2, \ldots, X_n\}$ is is complete sufficient statistics for μ.

(b) Let $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2$, show that and $X_{(1)}$ and S^2 are independent

Question.2 (**8+7=15-Points**)

(a) Let $X_1, X_2, \ldots, X_n$ be *iid* from PDF $f(x;\theta) = \begin{cases} \frac{x}{\theta} e^{-\frac{x^2}{2\theta}} & x > 0 \\ 0 & ,\text{Otherwise} \end{cases}$, $\theta > 0$. Show that $\sum_{i=1}^{n} X_i^2$ is a minimal sufficient statistics for θ

(b) $P\{X = x\} = \begin{cases} p & , if X = 0 \\ 3p & , if X = 1 \\ 1 - 4p & , if X = 2 \end{cases}$, Show that the family of distributions of X is not complete.

Question.3 (**10+2=12-Points**)

Let $X_1, X_2, \ldots, X_n$ be *iid* from $N(\mu, \sigma^2)$, and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$.

(a) Find $E\{S^r\}$, where $r > 0$ is a real number.

(b) Obtain an unbiased estimator for σ^r

Question.4 (**14+8=22-Points**)

Let $X_1, X_2, \ldots, X_n$ be *iid* from $b(1, \theta)$, $\theta \in (0, 1)$. Assume that k is a known positive integer number.

(a) Find the UMVUE of $\psi(\theta) = (1 - \theta)^k$

(b) Find the FCR lower bound for the estimator in part (a).

Question.5 (**8-Points**)

Let $X_1, X_2, \ldots, X_m$ be *iid* from $b(n, p)$. Find the method of moments estimators for (n, p).

Question .6 **(5+3=8-Points)**

Let $X_1, X_2, \ldots, X_n$ be *iid* RVs with PDF $f_\theta(x) = \theta(1-x)^{\theta-1}$, $0 < x < 1$, $\theta > 1$

(a) Find the MLE for θ.

(b) Find the MLE for $P_\theta\{X \leq \frac{1}{2}\}$.

Question .7 **(6+2+7=15-Points)**

Let $X_1, X_2, \ldots, X_n$ be a random sample from $P(\lambda)$. For estimating λ, using the quadratic error loss function, an a priori distribution over $\Theta = (0, \infty)$ given by the PDF: $\pi(\lambda) = \begin{cases} e^{-\lambda} & , \lambda > 0 \\ 0 & , \text{Otherwise} \end{cases}$ is used.

(a) Find the posterior distribution $h(\lambda|\boldsymbol{x})$.

(b) Find the Bayes's estimator of λ

(c) If the loss error function given by $L(\lambda, \delta) = \frac{(\lambda - \delta)^2}{\lambda}$. Find the Bayes's estimator of λ.