King Fahd University of Petroleum and Minerals Department of Mathematics and Statistics STAT-361 Operations Research I ¹ Final Exam

Four Problems, May 18 $^{th},\,2016^{\ 2}$

Problem 1 (30 pts)

Consider the following linear program (P):

$$\begin{aligned} & \min_{x_1, x_2} & & 3x_1 + 4x_2 \\ & \text{s.t.} & & 2x_1 + 3x_2 \geq 5, \\ & & & 5x_1 + 2x_2 \geq 7, \\ & & & x_1, x_2 \geq 0. \end{aligned}$$

(a) Solve the linear program (P) graphically. (10 points)

¹Dr. Slim Belhaiza (c)

²This is an open book exam. The exam lasts 120 minutes.

(b) Formulate the dual linear program (D) associated to the program (P). (5 points)

(c) Solve the linear program (D) graphically. (10 points)

Problem 2 (25 Points)

Consider the following transportation problem. The problem involves 3 plants supplying 4 customer zones. The following table 1 displays the unit transportation costs, the supplies and the demands. Find the optimal solution to this problem.

$\overline{\text{Demand Nodes} \rightarrow}$	1	2	3	4	
Supply Nodes ↓			Costs		Offer
1	3	3	3	6	1000
2	1	4	2	5	800
3	5	2	5	4	600
Demand	500	700	500	700	

Table 1: Data for problem 2

You can use the following tables. You have to use **Vogel** initialization method, otherwise you are subject to -5 points.

Interation 1

I	Demand Nodes	1	2	3	4	
	1					1000
	2					800
	3					600
-	Demand	500	700	500	700	

Interation 2

Demand Nodes	1	2	3	4	
1					1000
2					800
3					600
Demand	500	700	500	700	

Interation 3

Demand Nodes	1	2	3	4	
1					1000
2					800
3					600
Demand	500	700	500	700	

Interation 4

Demand Nodes	1	2	3	4	
1					1000
2					800
3					600
Demand	500	700	500	700	

Interation 5

Demand Nodes	1	2	3	4	
1					1000
2					800
3					600
Demand	500	700	500	700	

Problem 3 (25 pts)

Consider the following linear program:

$$\begin{aligned} \max_{x_1, x_2, x_3} & x_1 + 4x_2 + 3x_3 \\ \text{s.t.} & x_1 + x_2 + x_3 \leq 7, \\ & x_1 + 2x_2 + x_3 \geq 4, \\ & x_1 + x_2 + 4x_3 \geq 5, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

(a) Solve the linear program using the Primal Simplex algorithm.(12 pts)

(b) Solve the linear program using the Dual Simplex algorithm. (13 pts) $\,$

Problem 4 (25 pts)

Consider the following project scheduling problem detailed in table 2.

Tasks	Condition	Duration (days)
a	_	4
b	_	5
\mathbf{c}	after b	4
d	after a	3
e	after c and d	3
f	after d	4
g	after e	5
h	after e and f	5
i	after g	5
k	after h and i	3

Table 2: Data for problem 4

⁽i).Draw the graph representing the interdependence between the tasks of the project.

⁽¹⁰ pts)

⁽ii). Find the shortest possible duration of the project. (10 pts)

⁽iii). Find the critical tasks and the critical tasks. (5 pts)