

## STAT-319 Formula Sheet for Final Exam Term 152

### Chapter 2

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A \cap B^c) + P(A \cap B) + P(A^c \cap B)$$

$$P(A \cup B) = 1 - P(A \cup B)^c = 1 - P(A^c \cap B^c) \quad \text{and} \quad P(A \cap B)^c = P(A^c \cup B^c)$$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0 \quad \text{and} \quad P(A \cap B) = P(A) \cdot P(B | A) = P(A | B) \cdot P(B)$$

$$P(B_i | A) = \frac{P(A | B_i) \cdot P(B_i)}{\sum_{i=1}^k P(A | B_i) \cdot P(B_i)}, P(A) \neq 0$$

### Chapter 3

$$\mu = E(X) = \sum x f(x); \quad E(X^2) = \sum x^2 f(x) \quad \text{and} \quad \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2$$

$$f(x) = \frac{1}{n}; \quad x = x_1, x_2, \dots, x_n; \quad \mu = \frac{x_n + x_1}{2}; \quad \sigma^2 = \frac{(x_n - x_1 + 1)^2 - 1}{12} \quad (\text{for consecutive integers from } x_1 \text{ to } x_n)$$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}; \quad x = 0, 1, \dots, n; \quad \mu = np; \quad \sigma^2 = np(1-p)$$

$$f(x) = p(1-p)^{x-1}; \quad x = 1, 2, \dots; \quad \mu = 1/p; \quad \sigma^2 = (1-p)/p^2$$

$$f(x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}; \quad x = \max\{0, n + K - N\} \text{ to } \min\{K, n\}; \quad \mu = np; \quad \sigma^2 = np(1-p) \frac{N-n}{N-1}; \quad p = \frac{K}{N}$$

$$f(x) = \frac{e^{-\lambda t} (\lambda t)^x}{x!}; \quad x = 0, 1, \dots; \quad \mu = \lambda t; \quad \sigma^2 = \lambda t$$

### Chapter 4

$$F(b) = P(X \leq b) = \int_{-\infty}^b f(x) dx \quad \text{and} \quad P(a < X < b) = \int_a^b f(x) dx = F(b) - F(a)$$

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx; \quad E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx \quad \text{and} \quad \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2$$

$$f(x) = \frac{1}{x_n - x_1}; \quad x_1 \leq x \leq x_n; \quad \mu = \frac{x_n + x_1}{2}; \quad \sigma^2 = \frac{(x_n - x_1)^2}{12}$$

$$f(x) = \lambda e^{-\lambda x}; \quad x > 0; \quad \mu = \frac{1}{\lambda}; \quad \sigma^2 = \frac{1}{\lambda^2}$$

$$X \sim N(\mu, \sigma^2); \quad Z = \frac{X - \mu}{\sigma} \sim N(0, 1); \quad Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

## Chapter 6

Sample mean :  $\bar{X} = \frac{\sum X}{n}$ ; Sample variance:  $S^2 = \frac{\sum (X - \bar{X})^2}{n-1} = \frac{\sum X^2 - \frac{1}{n}(\sum X)^2}{n-1}$

Locating percentiles ( $P_\alpha$ ):  $R_\alpha = \frac{\alpha}{100}(n+1) = id$  and  $P_\alpha = X_{(i)} + d(X_{(i+1)} - X_{(i)})$

Coefficient of Variation:  $CV = s / \bar{X}$ ; Coefficient of skewness:  $CS = \frac{3(\bar{X} - \tilde{X})}{s}$

## Chapter 7

$X \sim N(\mu, \sigma^2) \Rightarrow Z = \frac{X - \mu}{\sigma} \sim N(0,1)$ ;  $\bar{X} \sim N(\mu, \sigma^2/n) \Rightarrow Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$

$\mu_{\bar{X}} = \mu$ ,  $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ ,  $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$ ,  $\sigma_{\bar{X}_1 - \bar{X}_2}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

## Chapters 8-10

Confidence Interval	Test Statistic
$\bar{x} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ and $n \geq \left(\frac{\sigma Z_{\frac{\alpha}{2}}}{e}\right)^2$	$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
$\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}$	$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
$(\bar{x}_1 - \bar{x}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	$Z = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
$(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}, n_1+n_2-2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$	$T = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ where $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$
$(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}, v} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$T = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ where $v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$
$\bar{d} \pm t_{\frac{\alpha}{2}, n-1} \frac{s_d}{\sqrt{n}}$	$T = \frac{\bar{d} - d_0}{s_d/\sqrt{n}}$
$\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ and $n \geq \frac{Z_{\frac{\alpha}{2}}^2 [p(1-p)]}{e^2}$	$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
$(\hat{p}_1 - \hat{p}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p}) \left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$ where $\hat{p} = \frac{x_1+x_2}{n_1+n_2} = \frac{n_1\hat{p}_1+n_2\hat{p}_2}{n_1+n_2}$

## Chapter 11

$$s_{xx} = \sum x^2 - \frac{1}{n}(\sum x)^2, \quad s_{yy} = \sum y^2 - \frac{1}{n}(\sum y)^2, \quad s_{xy} = \sum xy - \frac{1}{n}(\sum y)(\sum x)$$

$$Y = \beta_0 + \beta_1 X + \epsilon, \quad \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X, \quad e = Y - \hat{Y}, \quad \hat{\beta}_1 = \frac{s_{xy}}{s_{xx}}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$SST = SSR + SSE, \quad SSR = \hat{\beta}_1 s_{xy}, \quad SST = s_{yy},$$

$$SSE = SST - SSR, \quad MSE = \frac{SSE}{n-2}, \quad MSE = \widehat{\sigma^2},$$

$$r = \frac{s_{xy}}{\sqrt{s_{xx}s_{yy}}} = \hat{\beta}_1 \sqrt{\frac{s_{xx}}{s_{yy}}} \quad \text{and} \quad R^2 = \frac{SSR}{SST}$$

$\hat{\beta}_0 \pm t_{\frac{\alpha}{2}, n-2} \sqrt{MSE \left[ \frac{1}{n} + \frac{\bar{x}^2}{s_{xx}} \right]}$	$T = \frac{\hat{\beta}_0 - \beta_0}{\sqrt{MSE \left[ \frac{1}{n} + \frac{\bar{x}^2}{s_{xx}} \right]}}$
$\hat{\beta}_1 \pm t_{\frac{\alpha}{2}, n-2} \sqrt{\frac{MSE}{s_{xx}}}$	$T = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\frac{MSE}{s_{xx}}}}$
$\hat{y}_0 \pm t_{\frac{\alpha}{2}, n-2} \sqrt{MSE \left[ 1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{s_{xx}} \right]}$	
$\hat{y}_0 \pm t_{\frac{\alpha}{2}, n-2} \sqrt{MSE \left[ \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{s_{xx}} \right]}$	