King Fahd University of Petroleum & Minerals

Mathematics & Statistics Department

STAT 319: Probability & Statistics for Engineers & Scientists

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Term 152

First Major Exam

Saturday 20/02/2016

6:30 – 8:00 PM

Please circle your instructor name:

Al-Momani Al-Sawi Riaz Saleh Samuh

Std. Name: Key Std. ID: Serial No.:

Question No.	Full Mark	Marks Obtained
1.	8	
2.	11	
3.	13	
4.	9	
5.	9	
Total	**50**	

Q1]... [8 points] Find the error(s) in each of the following statements:

1. (2 points) The probabilities that a batch of 4 computers will contain 0, 1, 2, 3, or 4 defective computers are 0.547, 0.3562, 0.087, 0.0094 and 0.0005, respectively.

(+1) ✓ The Probabilitis don't sum to 1.

(+1) $0.547 + 0.3562 + 0.087 + 0.0094 + 0.0005 = 1.0001 > 1$.

2. (2 points) The probability that you will miss the idea of the next class is 0.4 and the probability that you will not miss the idea of the next class is 0.5.

(+1) ✓ The two Probabilities of complementary events don't sum to 1.

(+1) $0.4 + 0.5 = 0.9 \neq 1$.

3. (2 points) The probabilities that a person will make 0, 1, 2, 3, 4 calls in a day are 0.21, 0.15, −0.43, 0.25, 0.46, respectively.

(+1) ✓ Probability can't be negative.

(+1) $P(X=2) = -0.43 < 0$.

4. (2 points) A businessman claims that the probability he will get contract A is 0.15 and will get contract B is 0.2. Furthermore, he claims that the probability of getting A or B is 0.5.

(+1) ✓ $P(A \cup B) \leq P(A) + P(B)$

(+1) $0.5 \nleq 0.35$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0 \leq P(A \cap B) \leq 1$

Q2]...[11 points] The thickness of wood paneling (in inches) that a customer orders is a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & \text{for } x < \frac{1}{8} \\ 0.2 & \text{for } \frac{1}{8} \le x < \frac{1}{4} \\ 0.9 & \text{for } \frac{1}{4} \le x < \frac{3}{8} \\ 1 & \text{for } x \ge \frac{3}{8} \end{cases},$$

Determine the following:

1. (2 points) $P(X \le \frac{1}{18})$.

+1 $= F(\frac{1}{18})$

+1 $= \boxed{\text{Zero}}$.

2. (3 points) $P(X > \frac{1}{4})$.

+1 $= 1 - P(X \le \frac{1}{4}) = 1 - F(\frac{1}{4})$

+1 $= 1 - 0.9 = \boxed{0.1}$ +1

3. (3 points) $P\left(\frac{1}{4} \le x < \frac{3}{8}\right)$.

+1 $= P(X = \frac{1}{4})$

+1 $= 0.9 - 0.2$

+1 $= \boxed{0.7}$

4. (3 points) Obtain the probability mass function of X.

x	$\frac{1}{8}$	$\frac{1}{4}$	$3/8$	Tot.
$P(x)$	0.2	0.7	0.1	1

+2 for x-values

+1 for the probabilities.

Q3]... **[13 points]** The probability that your call to a service line is answered in less than 30 seconds is 0.75. Assume that your calls are independent.

1. (4 points) If you call 20 times, what is the probability that at least 2 calls are answered in less than 30 seconds?

$$X \sim Bin(20, 0.75)$$

+1 $$P(X \geq 2) = 1 - P(X \leq 1) = 1 - [P(0) + P(1)]$$ +1

$$= 1 - [(0.25)^{20} + (20)(0.75)(0.25)^{19}]$$ +1

$$\simeq \boxed{1}$$ +1

2. (3 points) If you call 20 times, what is the mean number of calls that are answered in less than 30 seconds?

$$X \sim Bin(20, 0.75)$$

$$E(X) = np$$ +1

$$= (20)(0.75)$$ +1

$$= \boxed{15}$$ +1

3. (3 points) What is the probability that you must call four times to obtain the first answer in less than 30 seconds?

$$Y \sim Geo(0.75)$$

+1 $$P(Y = 4) = (0.25)^3(0.75)$$ +1

$$= \boxed{0.01171875}$$ +1

4. (3 points) What is the mean number of calls until you are answered in less than 30 seconds?

$$E(Y) = \frac{1}{p}$$ +1

$$= \frac{1}{0.75}$$ +1

$$\simeq \boxed{1.33}$$ +1

Q4]... **[9 points]** A box contains 10 fuses. Eight are rated at 10 Amperes and the others rated at 15 Amperes.

A B

1. Two fuses are selected at random without replacement.

   (a) (3 points) What is the probability that the second fuse rate at 15 Amperes?

$$+1 \quad P(A_1 B_2) + P(B_1 B_2) = \frac{8}{10} \cdot \frac{2}{9} + \frac{2}{10} \cdot \frac{1}{9} \quad +1$$

$$= \frac{8}{45} + \frac{1}{45} = \boxed{\frac{9}{45}} \quad +1$$

   (b) (3 points) What is the probability that the second fuse rate at 15 Amperes, given that the first fuse rated at 10 Amperes?

$$P(B_2 | A_1) = \frac{P(A_1 \cap B_2)}{P(A_1)} \quad +1$$

$$= \frac{\frac{8}{10} \cdot \frac{2}{9}}{8/10} \quad +1$$

$$= \boxed{\frac{2}{9}} \quad +1$$

2. (3 points) Fuses are randomly selected from the box, one by one with replacement, until a 15 Amperes fuse is selected. What is the probability that a total of two fuses are selected from the box?

$$+1 \quad P(A_1 B_2) = \frac{8}{10} \cdot \frac{2}{10} \quad +1$$

$$= \boxed{0.16} \quad +1$$

Q5]...[9 points] A university student frequents one of two coffee houses on campus, choosing Costa Coffee 70% of the time and Starbucks 30% of the time. Regardless of where he goes, he buys a decaffeinated coffee on 60% of his visits.

A B D

$P(A) = 0.7$, $P(B) = 0.3$, $P(D) = 0.6$

1. (3 points) The next time he goes into a coffee house on campus, what is the probability that he goes to Costa Coffee and orders a decaffeinated coffee?

$$P(A \cap D) = P(A)P(D|A) \quad +1$$

$$+1 \quad = P(A) \cdot P(D) \qquad D \,\&\, A \text{ are indep.}$$

$$+\tfrac{1}{2} \quad = (0.7)(0.6)$$

$$+\tfrac{1}{2} \quad = \boxed{0.42}$$

2. (4 points) What is the probability that he goes to either of the two coffee houses and order a regular coffee?

$$+1 \quad P((A \cup B) \cap D^c) = P(A \cap D^c) + P(B \cap D^c) \quad \because A \,\&\, B \text{ mutually exclu.}$$

$$+1 \quad = P(A)P(D^c) + P(B)P(D^c) \quad \because \text{independence.}$$

$$+1 \quad = (0.7)(0.4) + (0.3)(0.4)$$

$$+1 \quad = 0.28 + 0.12 = \boxed{0.40}$$

3. (2 points) If he goes into a coffee house and orders a decaffeinated coffee, what is the probability that he is at Starbucks?

$$+1 \quad P(B|D) = P(B) \quad \because \text{independence.}$$

$$+1 \quad = \boxed{0.3}$$

GOOD LUCK