KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 301: Introduction to Probability Theory
Semester 152
Third Major Exam
Thursday April 28, 2016
3:45 − 5:45 pm

Name: ID #:

Question No	Full Marks	Marks Obtained
1	**12**	
2	**11**	
3	**10**	
4	**10**	
5	**10**	
Total	**53**	

Q.No.1:- (2+5+5 = 12 points) Suppose that the two continuous random variables X and Y follow the following joint probability density function:

$f(x, y) = k(3x - y);$ $\quad 0 < x < 1, \quad 0 < y < 1$

(a) Find the value of k.

(b) Find $P(X < 0.4 \mid Y = 0.5)$

(c) Find ρ_{XY} (the correlation coefficient between X and Y).

Q.No.2:- (4+7 = 11 points)

(a) Suppose that a continuous random variable X follows the following distribution:

$$f(x) = \frac{1}{\sqrt{32\pi}} e^{-\frac{1}{32}(X^2+100-20X)}; \qquad -\infty < x < \infty$$

Find the moment generating function of X.

(b) Let $X_1, X_2, \dots \dots, X_n$ be a random sample from a distribution with mean μ_X and variance σ_X^2. Define $S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n+1}$ where $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ is the sample mean. Find $E(S^2)$.

Q.No.3:- (5+5 = 10 points)Suppose that X_1 and X_2 are independent random variables having a common mean μ. Suppose also that $\text{Var}(X_1) = \sigma_1^2$ and $\text{Var}(X_2) = \sigma_2^2$. Define a new variable $Z = \alpha X_1 + (1-\alpha)X_2$ where α is a constant.

(a) Derive the variance of this new variable Z i.e. $\text{Var}(Z)$.

(b) Find the value of α for which the $\text{Var}(Z)$ is minimum.

Q.No.4:- (5+5 = 10 points)

(a) Let X_1 and X_2 follow the following joint distribution:

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left[x_1^2 + x_2^2 - 2\rho x_1 x_2\right]}; \qquad -\infty < x_1 < \infty, \quad -\infty < x_2 < \infty$$

Find the marginal distribution of X_2 and show that it is a valid probability density function.

(b) Let Y_1, Y_2 and Y_3 be three independent standard normal random variables. If $Z_1 = Y_1 + Y_2$, $Z_2 = Y_1 + Y_3$ and $Z_3 = Y_2 + Y_3$, compute (and simplify) the joint distribution of Z_1, Z_2 and Z_3.

Q.No.5:- (10 points) Suppose that 3 balls are chosen without replacement from an urn containing 5 white and 2 black balls. Let X denotes the number of white balls chosen and Y represents the number of black balls chosen.

Fill the following table of Joint Probability Mass Function.

<table>
<tr><td colspan="2" rowspan="2">Joint Probability Mass Function</td><td colspan="4">Y</td><td rowspan="2">Marginal of X</td></tr>
<tr><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td rowspan="4">X</td><td>0</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Marginal of Y</td><td></td><td></td><td></td><td></td><td></td></tr>
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