KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 301: Introduction to Probability Theory
Semester 152
Second Major Exam
Wednesday March 30, 2016
8:30 − 10:30 pm

Name: ID #:

Question No	Full Marks	Marks Obtained
1	**10**	
2	**12**	
3	**08**	
4	**15**	
5	**10**	
Total	**55**	

Q.No.1:- (5+5 = 10 points)

(a) Let X be a Binomial random variable denoting the number of customers who buy a ticket (out of first $n = 100$) at the office of Saudi Airline on a given day. The probability that a specific customer buys a ticket is $p = 0.7$.

Define a new variable $Y = 7 + 2X$. Find $E[Y]$ and $E[Y^2]$.

(b) Suppose $P(X = a) = p = 1 - P(X = b)$. Show that $Var(X) = p(1 - p)(a - b)$.

Q.No.2:- (6+6 = 12 points)

(a) In a specific city of America, there are 46% families who own a car. 30% of the families have less than or equal to 3 family members. Moreover, there are 28% families who own a car and they have more than 3 family members. If a family is randomly selected from that city, find the conditional probability that

(i) the family owns a car given that they have less than or equal to 3 family members.

(ii) the family has more than 3 members given that they do not own a car.

(b) In term 152, STAT-301 has 9 actuaries and 1 other student where as STAT-435 has 3 actuaries and 5 other students. We throw a die. If the outcome is an odd number, we go to STAT-301 class and pick a student, whereas if the outcome is an even number, we go to STAT-435 class and select a student. Suppose that the picked student is an actuary, what is the probability that the die landed on an even number?

Q.No.3:- (3+2+3 = 8 points) Suppose that the distribution function of X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ x/4 & 0 \le x < 1 \\ \dfrac{1}{2} + \dfrac{x-1}{4} & 1 \le x < 2 \\ 11/12 & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

(a) Find $P\{X = i\}$, $i = 1,2,3$.

(b) Find the probability mass function of X.

(c) Find $P\left\{\frac{1}{2} < X \le \frac{3}{2}\right\}$.

Q.No.4:- (9+6 = 15 points)

(a) Derive the mean and variance of Bernoulli Distribution using moment generating function.

(b) Let X denotes the number of cars passing through a checkpoint in a minute such that X can be monitored by an approximate Poisson process with average no. of cars passing through that checkpoint $\lambda = 3.5$ per minute. If Y denotes the waiting time till first car passes through that checkpoint, derive the probability density function of Y.

Q.No.5:- (4+6 = 10 points) The length of the tube rods (in cm) is normally distributed with mean 80 cm and standard deviation 0.5 cm. If the length of the tube rod is between 79.4 cm and 80.8 cm, it is considered as non-defective. Otherwise, the rod is declared defective.

(a) What is the probability that a randomly selected tube rod is defective?

(b) If we have a lot of 2500 tube rods, what is the probability that at least 2100 are non-defective?

With the Best Wishes