King Fahd University of Petroleum & Minerals
Department of Math. & Stat.

Math 568 - Final Exam (152)   Time: 3 hours 00 mns

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Name: ID #

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Problem 1	/5
Problem 2	/10
Problem 3	/8
Problem 4	/5
Problem 5	/8
Problem 6	/9
Total	/45

Problem # 1. (5 marks)

a. Check that $u(x,y) = 3yx^2 + xy - y^3 + 3$ is harmonic

b. Compute

$$\int\int_{\Omega} u(x,y)dxdy,$$

where

$$\Omega = \{(x,y)/x^2 + y^2 < 2\}$$

Hint: Do not forget the mean value property

Problem # 2. (10 marks) Solve the problem

$$\begin{cases} \Delta u(x,t) - ku(x,y) = 0, & \text{in } (0,\pi)\times(0,\pi) \\ u(\pi,y) = 2\sin 3y, & u(0,y) = u(x,0) = u(x,\pi) = 0 \end{cases}$$

where $k > 0$ is a constant.

Problem # 3. (8 marks) Solve the problem

$$\begin{cases} u_t(x,t) - 2\Delta u(x,t) = 0, & \text{in } \mathbb{R}^2 \times (0,+\infty) \\ u(x,0) = x_1 x_2, & \text{in } \mathbb{R}^2 \end{cases}$$

Problem # 4. (5 marks) Let $u(x,t)$ be the solution of the problem

$$\begin{cases} u_t(x,t) - u_{xx}(x,t) = 0, & \text{in } (0,\pi)\times(0,+\infty) \\ u(0,t) = u(\pi,t) = 0, & t \geq 0 \\ u(x,0) = \sin^2(x), \quad 0 \leq x \leq \pi \end{cases}$$

Use the maximum principle to show that

$$0 \leq u(x,t) \leq e^{-t}\sin x, \text{ in } (0,\pi)\times(0,+\infty)$$

Hint: let $v(x,t) = e^{-t}\sin t$ and compare $v(x,0)$ and $u(x,0)$.

Problem # 5. (7 marks) Solve the initial-boundary value problem

$$(P3)\quad \begin{cases} u_{tt}(x,t) - u_{xx}(x,t) = 0, & \text{in } (0,+\infty)\times(0,+\infty) \\ u(0,t) = 3 - t, & t \geq 0 \\ u(x,0) = 3 + \sin x, & \text{in } (0,+\infty) \\ u_t(x,0) = -1 + x^2, & \text{in } (0,+\infty) \end{cases}$$

Problem # 6. (10 marks) Solve

$$\begin{cases} u_{tt}(x,t) - 16\Delta u(x,t) = 0, & \mathbb{R}^3 \times (0,+\infty) \\ u(x,0) = 1 + x_1 x_2, \qquad u_t(x,0) = x_3^2, & \text{in } \Omega \end{cases}$$