King Fahd University of Petroleum & Minerals
Department of Math. & Stat.

Exam I - Math 568 (152) Time: 2 hours 15 mns

==========

Name: ID #

==========

Please show all work. No credit for a result without work

Problem 1	/7
Problem 2	/10
Problem 3	/10
Problem 4	/8
Total	/35

Problem # 1. (7 marks) Find the function $u(x, y, t)$ that solves the problem

$$u_t - (y^2 - 1)u_x = 0,$$
$$u(x, y, 0) = e^{y-x^2}$$

Problem # 2. (10 marks) Use the characteristic method to find **two** solutions of

$$u_x^2 + u_y^2 = 4u, \qquad u(x,x) = 2x^2 \tag{1}$$

Problem # 3. (10 marks) Let

$$u_{xx} - 6u_{xy} + 9u_{yy} = xy^2 \tag{2}$$

a. Show that (2) is parabolic
b. By a convenient change of variable, reduce it to

$$w_{\eta\eta} = \frac{1}{27}(\xi - \eta)\eta^2$$

c. Find the solution $u(x, y)$, of (2), and its domain of definition, if $u(x, 0) = \sin x$ and $u_y(x, 0) = \cos x$, $x \in \mathrm{R}$.

Problem # 4. (8 marks) Show that the function defined in $\Omega = (0,1) \times (0,1)$ and given by

$$u(x,t) = \begin{cases} t & t > x \\ 2x - t & t \leq x \end{cases}$$

is a weak solution of

$$u_t + u_x = 1.$$