King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
Math 550: Linear Algebra
Second Exam, Spring Semester 152 (120 minutes)
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Q1. (20 points) Let V be a non-zero vector space. Show that:
(a) If f is a non-zero functional on V, then the null-space of f is a hyperspace in V.
(b) Every hyperspace in V is the null-space of some non-zero functional on V.

Q2. (25 points) Consider $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$, where

$$T(x, y, z) = (x - 4y - 4z, 8x - 11y - 8z, -8x + 8y + 5z).$$

(a) Show that T is diagonalizable.
(b) Find a basis β such that $[T]_\beta$ is a diagonal matrix.

Q3. (25 points) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be given by

$$T(x, y, z) = (3x + y - z, 2x + 2y - z, 2x + 2y).$$

Write $T = D + N$, where D is a diagonalizable operator on $\mathbb{R}^3$ and N is a nilpotent operator on $\mathbb{R}^3$, such that $DN = ND$. Find the matrices of D and N w.r.t. the standard basis of $\mathbb{R}^3$.

Q4. (20 points) Let T be a linear operator on a finite dimensional vector space $V = W_1 \oplus W_2$, such that both W_1 and W_2 are invariant under T. Set $T_1 = T_{|W_1}$, $T_2 = T_{|W_2}$ and show that:
(a) $\text{char}(T) = \text{char}(T_1)\ \text{char}(T_2)$.
(b) $\min(T) = \min(T_1)\ \min(T_2)$.

Q5. (10 points) Prove or disprove:
(a) If A is a matrix with real entries such that $A^t A = 0$, then $A = 0$.
(b) If a linear operator T has a cyclic vector, then T^2 has a cyclic vector.

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