King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 531 (Real Analysis) **Term 152**

Exam 1: February, 28, 2016 **Time allowed: 2hrs**

Question Number	Marks	Maximum Points
1		2
2		2
3		2
4		2
5		2
6		2
7		3
8		2
9		3
Total		20

Name of the student:

ID Number:

(Q1) Prove that Lebesgue outer measure m^* is countably subadditive.
Use this result to evaluate $m^*(Q)$ where Q is the set of rational numbers.

(Q2) Let A be any set of real numbers and $\varepsilon > 0$. Then show that there is an open set O such that $A \subseteq O$ and $m^*(O) < m^*(A) + \varepsilon$.
Under what condition on the set O, the above inequality becomes $m^*(A) = m^*(O)$ and how?

(Q3) Prove that for a, b in $\mathbb{R}$, the interval (a, b) is a Lebesgue measurable set. Use this result to show that any open set in $(\mathbb{R}, |\cdot|)$ is measurable.

(Q4) Show that the Lebesgue measure m is translation invariant.

(Q5) (a) Define a Borel set. Show that every Borel set is Lebesgue measurable.

(b) Use an appropriate result to find Borel measure of any countable set of numbers.

(Q6) Let P be the usual non-Lebesgue measurable subset of $A = [0, 1)$. If E is measurable and $E \subset P$, then show that $m(E) = 0$.

(Q7) Prove or disprove:

(a) Any subset of a measurable set is measurable.

(b) The outer measure of a non-measurable set is positive.

(c) The Cantor set C is a Borel set.

(Q8) Show by means of an example that if the set A has Lebesgue measure zero, then its closure and boundary need not have measure zero.

(Q9) For the sequence $E_n = [0,1) \cup [n, n+1)$, check whether or not the following equation holds:

$$\lim_{n\to\infty} m(E_n) = m\left(\lim_{n\to\infty} E_n\right)$$

where m stands for Lebesgue measure.