KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

Department of Mathematics & Statistics

SYLLABUS

MATH 442 (152)

Instructor: Prof. Bilal Chanane

Title: Calculus of Variations and Optimal Control
Credit: 3-0-3
Textbook: Optimal Control and the Calculus of Variations, E. R. Pinch, Oxford university Press, 1995
References:

(i) Calculus of Variations, M. Gelfand and S. V. Fomin
(ii) Variational Calculus and Optimal Control, 2nd Edition, J. L. Troutman, Springer 1996
(iii) Calculus of Variations and Optimal Control Theory: A Concise Introduction, Daniel Liberzon, Princeton University Press, 2011

Description: Introduction to the calculus of variations. Euler-Lagrange, Weierstrass, Legendre and Jacobi necessary conditions. Formulation of optimal control problems. Bolza, Mayer and Lagrange formulations. Variational approach to optimal control. Pontryagin maximum principle.
Prerequisite: MATH 202

Learning outcomes:

At the end of this course, the student should be able to

1. Formulate a variational problem,
2. Derive from first principles necessary conditions for an extremum in the simplest case and the case where the functional depends upon several variables and their higher order derivatives,
3. Solve the Euler-Lagrange equation in specific special cases.
4. Formulate an optimal control problem
5. Work with the Bolza, Mayer and Lagrange formulations
6. Use the variational approach to optimal control
7. State and work with the Pontryagin maximum principle
8. Use Dynamic programming in continuous time: Hamilton-Jacobi-Bellman equation.
9. Solve the linear quadratic regulator problem

Week#	Date	Topic
1	Jan. 17-21	Finite and infinite dimensional optimization, Brachistochrome and isoperimeter problems
2	Jan. 24-28	Function spaces, functionals
3	Jan.31-Feb. 4	Variation of a functional and necessary conditions of an extremum The simplest variational problem and Euler-Lagrange equation
4	Feb. 7-11	The case of several derivatives The variational derivative
5	Feb. 14-18	Invariance of Euler-Lagrange equation Fixed point problem for n unknown functions
		Major Exam I
6	Feb. 21-25	Variational problem in parametric form Functional depending on high order derivatives
7	Feb. 28-Mar.3	Cont. Variational problems with subsidiary derivatives
8	Mar. 6-10	The general variation of a functional End points lying on two different curves
		Mid Term Vacation March 11-19, 2016
9	Mar.20-24	Broken extremals, the Weirstrass-Erdmann conditions Statement of the optimal control problem
		Major Exam II
10	Mar.27-31	The Pontryagin maximum principle
11	Apr.3-7	Cont.
12	Apr.10-14	Dynamic programming in continuous time and the Hamilton-Jacobi-Bellman theory
13	Apr.17-21	Optimal control to target curves Time-optimal control
14	Apr.24-28	Fuel-optimal control Linear quadratic regulator
15	May 1-5	Review
		Final Exam

Teaching strategies to be used to develop these cognitive skills

- Homework assignments involving analytic development of the notions introduced.
- Project assignment involving topics not covered in great extent in class (e.g., broken extremals) and new topics such as Bernstein theorem, Noether'theorem)

Methods of assessment of students cognitive skills

- Major and final exams
- Homework
- Project report and presentation

Schedule of Assessment Tasks for Students During the Semester

Assessment	Assessment Task	Week due	Proportion of Final Assessment
1	Class activities (participation)	Weekly	5%
2	Homework	Weekly	10%
3	Major Exam I	Week 6	20%
4	Major Exam II	Week 10	20%
5	Final Exam	Week 16	30%
6	Projects	Monthly	15%