KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Math 442 Final Exam
June 18, 2016
Duration 3 h
Instructor: Dr. B. Chanane

Name:.....ID:.....

Exercise #1: Find the extremal(s) of $J[y] = \int_0^{\pi/2} \{y^2 - y'^2 - 2y\,[3\sin x + 2\cos x]\}\,dx$ subject to $y(0) = 1$ and $y(\pi/2) = \pi$.

Exercise #2: Find the extremal(s) of $J[y] = \int_0^a (1+x)^{-1} y'^2 dx$ satisfying $y(0) = 0$ and the point $(a, y(a))$ lies on the curve $z = \exp(x)$, $a > 0$.

Exercise #3: Starting from first principles, derive the Weierstrass-Erdmann conditions for the functional $J[y] = \int_a^b F(x, y, y')dx$ to have an extremum with one corner at $c \in (a, b)$ satisfying $y(a) = A$, $y(b) = B$, F being continuous and having continuous first order partial derivatives with respect to all of its arguments.

Exercise #4: Find the extremal(s) of the functional $J[y] = \int_0^1 (y'^2 + yy')dx$ where $y(0) = 0$, $y(1) = 2$ subject to the constraint $\int_0^1 y(x)dx = 1$.

Exercise #5 : The system $\frac{dx_1}{dt} = -x_1 + u$ is to be controlled from $x_1 = 0$ at $t_0 = 0$ to $x_1 = 1$ at some future time t_1 in such a manner that $J = \frac{1}{2}\int_{t_0}^{t_1} [x_1^2 + 3u^2]\, dt$ is minimized. Find the optimal control u and the corresponding x_1 and J.