KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Math 442 Major II Exam
April 12, 2016
Duration 2 h
Instructor: Dr. B. Chanane

Name:.....ID:.....

Exercise #1: (10pts) Write down the Weierstrass-Erdemann conditions for the functional $J[y] = \int_a^b F(x, y, y')dx$ to have an extremal with corner at $c \in (a, b)$, (DO NOT DERIVE THEM !).

Exercise #2 : (30pts) The system $\frac{dx_1}{dt} = -3x_1 + u$ is to be controlled from $x_1 = 0$ at $t_0 = 0$ to $x_1 = 1$ at some future time t_1 in such a manner that $J = \int_{t_0}^{t_1} u^2(t)dt$ is minimized. Use the maximum principle to obtain the optimal control u and the corresponding x_1 and J.

Exercise #3 : (30pts) The oscillatory system $\frac{dx_1}{dt} = x_2$, $\frac{dx_2}{dt} = -4x_1 + u$ is to be controlled, in the time interval $0 \leq t \leq \pi/2$, from $x_1 = 0$, $x_2 = 1$ in such a way that

$$J = (x_1(\pi/2))^2 + \int_0^{\pi/2} u^2 dt$$

is minimized. Find the optimal control and the corresponding value of J, Why the system is said to be oscillatory? .

Exercise #4 : (30pts) Consider the system $\frac{d^2x}{dt^2} = u$ where the control u satisfies $|u| \leq 2$.Describe the trajectories in phase space corresponding to the fastest motions from the given points $(2, 1)$ and $(-3, 2)$ to the origin $(0, 0)$. Draw these trajectories and specify the controls applied in each case.