King Fahd University of Petroleum and Minerals
deprtment of Mathematics and Statistics
Math 442 (Term 512)
Major Exam I (2hrs)

Name:.....ID:.....

NOTE: In Exercise 1 and 2, workout all of the details.
In Exercise 2 and 4, fisrt obtain the necessary conditions
as asked for both, then go back and solve the resulting problems.
A good presentation is a must.

Exercise 1 : (1a) **(15 pts)** Derive from first principles the necessary conditions for the functional

$$J[y] = \int_a^b F(x, y, y')dx$$

to have an extremum among twice continuously differentiable functions y satisfying $y(a) = A$.

(1b) **(15 pts)** Use (1a) to solve the following variational problem

$$\min_{y} J[y] = \int_0^1 \left\{ y^2 + y'^2 - 2xy \right\} dx \ , \ y(0) = 0$$

Exercise 2: (2a) **(15 pts)**Derive from first principles the necessary conditions for the functional

$$J[y] = \int_a^b F(x, y, y', y'')dx$$

to have an extremum among four times continuously differentiable functions y satisfying $y(a) = A_1$, $y'(a) = A_2$, $y(b) = B_1$, $y'(b) = B_2$.

(2b) **(15 pts)**Use (2a) to solve the following variational problem

$$\min_y J[y] = \int_0^1 \left[y^2 + y''^2\right] dx \ , \ y(0) = y'(0) = y(1) = 0, \ y'(1) = 1$$

Exercise 3: **(20 pts)** Obtain the necessary conditions for the variational problem below to have a solution

$$\min_y J[y] = \int_0^1 \left\{y^2 + 2y'^2\right\} dx \ , \ y(0) = 0, \ y(1) = 0, \ \int_0^1 \left\{y^2 + y'^2\right\} dx = 1$$

Exercise 4: **(20 pts)** Obtain the necessary conditions for the variational problem below to have a solution

$$\min_{y} J[y] = \int_0^1 \left\{ y^2 + z^2 + 2y'^2 \right\} dx \ , \ y(0) = 1, \ y(1) = 0, z(0) = 0, z(1) = 0, y^2 + z^2 + x^2 = 1$$