King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 430
Final Exam – 2015–2016 (152)
Allowed Time: 180 minutes

Name: ____

ID #: __________________________

Section #: __________ Serial Number: _______________

Instructions:

1. Write clearly and legibly. You may lose points for messy work.
2. **Show all your work**. No points for answers without justification.
3. **Calculators and Mobiles are not allowed.**

Problems:

Question #	Grade	Maximum Points
1		**12**
2		**12**
3		**10**
4		**12**
5		**12**
6		**14**
7		**16**
8		**12**
Total:		**100**

Q:1 (3 + 3 + 3 + 3 points) (a) If log (- e i) = a + ib, find a and b.
(b) Solve: 2 cosh z + sinh z = 2.
(c) Solve $z^2 = 4i$.
(d) Show that $i\pi$ is a period for the periodic function $f(z) =$ coth z.

Q:2 (6 + 6 points) (a) If $f(z) = e^x$ cos y + $iv(x, y)$ is an analytic function for any z. Find $v(x, y)$ and write $f(z)$ in terms of z.

(b) Evaluate $\int_C \text{Im}(\text{z} + \text{i})\ \text{dz}$, where C is the circular arc (in the first quadrant) along $|z| = 1$ from $z = 1$ to $z = i$.

Q:3 (10 points) Use Cauchy's integral formula to evaluate

$$\int_{\tau} \frac{z+1}{z^4 \ + \ 2iz^3} \, dz,$$

where τ is the circle $|z| = 1$.

Q:4 (12 points) Expand $f(z) = \dfrac{1}{1 - z}$ in a Taylor series with center $z_0 = 2i$. Also, find the circle of convergence of the Taylor series by ratio test.

Q:5 (8 + 4 points) (a) Expand $f(z) = \frac{1}{z\ (1-z)}$ in a Laurent series valid for $1 < |z-2| < 2$.

(b) Show that $z = 0$ is removable singularity of $f(z) = \dfrac{\sin 5z - 5z}{z^2}$

Q:6 (7 + 7 points)(a) State the Cauchy's residue theorem and use it to evaluate

$$\int_{|z|=2} \tan z \, dz.$$

(b) Evaluate $\int_0^{2\pi} \dfrac{1}{(2 + \cos\theta)^2} d\theta$.

Q:7 (8 + 8 points) (a) Let $|F(z)| \leq \frac{M}{R^k}$ for $z = Re^{i\theta}$, where $k > 0$ and M are constants. Prove that

$$lim_{R\to\infty} \int_\tau e^{imz} F(z) dz = 0,$$

where τ is the semicircular arc of radious R $(0 \leq \theta \leq \pi)$ and m ia constant.

(b) Evaluate $\int_0^\infty \dfrac{dx}{x^6 + 1}$.

Q:8 (2 + 6 + 4 points) (a) State the Argument principle.
(b) State and prove Rouche's theorem.
(c) Show that $z^3 + 9z + 27 = 0$ has no roots in $|z| < 2$.