King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 430 Exam 03
The Second Semester of 2015-2016 (152)
Time Allowed: 90 Minutes

Name: ______________________ ID#: ______________

Section/Instructor: ________________ Serial #: ______________

- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.

Question #	Marks	Maximum Marks
1		13
2		14
3		10
4		13
Total		50

Q1: (2 + 2 + 2 + 3 + 4 points) (a) State the Cauchy integral theorem.

(b) Find the directed parametrization of the line segment L joining the points $z_0 = i - 1$ to $z_1 = 2$ with parametrization interval $[\frac{1}{4}, \frac{3}{4}]$.

(c) Compute $\int_\tau \bar{z}^2 dz$ along the line segment joining two points $2 + 2i$ and 0.

(d) Evaluate $\int_{C_r} (z - z_0)^n dz, \ n \in I, \quad C_r : |z - z_0| = r$ transvered once in the counterclock-wise direction.

(e) State the ML-inequality and use it to find the upper bound of $\int_\tau \frac{e^{2z}}{z+1} dz$, where τ is the circle $|z| = 2$.

Q2: (8 + 6 points)(a) State and prove Cauchy's integral formula for an analytic function $f(z)$ in a simply connected domain D.

(b) Evaluate

(i) $\int_{|z-2|=2} \frac{e^{-z}}{(z+1)^2} dz$ (ii) $\int_{|z|=2} \frac{5z+7}{z^2+2z+3} dz$

Q3: (10 points) State the maximum modulus principle and use it to find the maximum value of $|(z - \frac{1}{2})(z + 1)|$ in the disk $|z| \leq 1$.

Q4: (3 + 3 + 3 + 4 points) (a) Using the raio test to find the domain in which convergence hold(s) for the series $\sum_{k=0}^{\infty} (z + 5i)^{2k} (k + 1)^2$.

(b) State the comparison test for the series of function.

(c) Prove that if the sequence $\{z_n\}_1^\infty$ converges, then $(z_n - z_{n-1}) \to 0$.

(d) Show that the sequence of functions $F_n(z) = \frac{z^n}{z^n - 3^n}, \quad n = 1, 2, \cdots$ converges to zero for $|z| < 3$ and to 1 for $|z| > 3$.