King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 430 Exam 02
The Second Semester of 2015-2016 (152)
Time Allowed: 90 Minutes

Name: _____ ID#: _____

Section/Instructor: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.

Question #	Marks	Maximum Marks
1		10
2		14
3		12
4		14
Total		50

Q1: (8 + 2 points) Let $z = x + iy$ and $f(z) = 4x + i\ 5y$.

(a) Show that $f(z)$ is differentiable only $x-$ axis and a vertical line $x = b$ for some real number b.

(b) Show that $f(z)$ is nowwhere analytic.

Q2: (6 + 2 + 6 points)(a) State the Cauchy-Riemann equations. Show that

$f(z) = e^{x^2-y^2}\ [cos(2xy) + i\ sin(2xy)]$ is entire and find its derivative.

(b) Verify that the function $u(x, y) = x^3-3xy^2-5y$ is harmonic in the entire complex plane.

(c) Find a harmonic conjugate of $u(x, y) = x^3 - 3xy^2 - 5y$.

Q3: (6 + 2 + 2 points) (a) Express $P_5(z) = z^5 - 1$ as a product of linear and quadratic factors.

(b) Prove that $cos(z) = 0$ iff $z = \frac{\pi}{2} + k\pi, \quad k \in I$.

(c) Find all numbers z (if any) such that $e^{iz} = 3$.

Q4: (4 + 3 + 4 + 3 points) (a) Determine the domain of analyticity for $f(z) = Log(4+i-z)$.

(b) Find all the values of $(1 + i)^3$.

(c) Derive the identity [write every step line by line]

$$sin^{-1}z = -i\ log[iz + (1 - z^2)^{\frac{1}{2}}].$$

(d) Find all values of z satisfying $sin(z) = 2$.