King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 430 Major Exam I
The Second Semester of 2015-2016 (152)
Time Allowed: 90 Minutes

Name: _____ ID#: _____

Section/Instructor: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.

Question #	Marks	Maximum Marks
1		4
2		4
3		5
4		4
5		4
6		5
7		4
8		5
9		5
10		4
11		6
Total		50

1 (4 points) Express $[\dfrac{1+i}{i-(1-2i)}]^2$ in terms of rectangular and polar forms.

(2) (4 points) If $z_1 = 2+i,\ z_2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}\ i$, then evaluate $z_1^3 - 3z_2^2 - 4Z_1 + 8$.

(3) (5 points) Prove that $||z_1| - |z_2|| \leq |z_1 - z_2|$.

(4) (4 points) Solve the following system for z_1 and z_2 and write your answer in standard form $a + ib$:

$$iz_1 - iz_2 = 2 + 10i$$
$$-z_1 + (1-i)z_2 = 3 - 5i.$$

(5) (4 points) Find the roots of $(-1+i)^{\frac{1}{3}}$ and locate them graphically.

(6) (5 points) (a) Define " Domain and open set " in the complex plane.
(b) Is the set $0 < |z-2| < 3$ a domain and open set? Give reason.

(7) (4 points)Let a set D be a domain set. Prove that a real valued function $U : D \to R$ is constant if $U_x = U_y = 0$ on D.

(8) (5 points) Find the image of the line $x = 1$ under the mapping $w = z^2$ and represent the mapping graphically.

(9) (5 points) Find the area of the rectangle whose vertices are the roots of the equation

$$z\bar{z}^3 + \bar{z}z^3 = 350,$$

where $Re(z)$ and $Im(z)$ are integers.

(10) (4 points) Use $\epsilon - \delta$ definition to show that $\lim_{z\to 1-i}\ (2+i)z = 3 - i$

(11) (6 points) Let complex numbers α and $\dfrac{1}{\bar{\alpha}}$ lie on the circles $|z - z_0| = r$ and $|z - z_0| = 2r$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2\ |z_0|^2 = r^2 + 2$, then find the value of $|\alpha|$.