King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Final Exam
The Second Semester of 2015-2016 (152)

Time Allowed: 180 Minutes

Name: ______________________________ ID#: ________________

Instructor: ________________________ Sec #: ________ Serial #: ________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		20
2		20
3		20
4		22
5		22
6		18
7		18
Total		140

Q:1 (20 points) Find temperature $u(x, t)$ in a rod of length 4 if initial temperature is

$f(x) = 4 - x$ and if ends at $x = 0$ and at $x = 4$ are insulated.

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Q:2 (20 points) Use separation of variables method to solve the Laplace's equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \quad 0<x<\pi, \ y>0,$$

subject to the boundary

$$\begin{aligned} u(0,t) &= 0, \quad u(\pi,t)=0, \quad y>0, \\ u(x,0) &= x, \quad 0<x<\pi. \end{aligned}$$

Also $u(x,y)$ is bounded as $y \to \infty$.

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Q:3 (20 points) Use Laplace transform to solve the problem

$$a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad x > 0,\ t > 0,$$

subject to the boundary and initial conditions

$$\begin{aligned} u\,(0,t) &= f(t), \quad \lim_{x\to\infty} u\,(x,\,t) = 0, \quad t > 0, \\ u\,(x,0) &= 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \quad x > 0, \end{aligned}$$

where $f(t) = \begin{cases} \cos t & 0 < t \le \frac{\pi}{2} \\ 0 & t > \frac{\pi}{2} \end{cases}$

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Q:4 (22 points) Find displacement $u(r,t)$ in a circular membrane of radius 2 if it is clamped along its circumference and if the membrane is given unit initial velocity and if its initial displacement is $f(r) = 1$. (Hint: Solve the following equation)

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial t^2},$$

Also solution $u(r,t)$ is bounded at $r = 0$.

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Q:5 (22 points) Find the steady-state temperature $u(r,\theta)$ in a sphere of unit radius by solving the problem

$$\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta}=0,\quad 0<r<1,\ 0<\theta<\pi,$$

subject to the boundary condition

$$u\,(1,\theta) \quad = \quad 1-\cos(2\theta), \quad 0<\theta<\pi.$$

Hint: $P_2(\cos\theta)=\frac{1}{4}(3\cos(2\theta)+1)$

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Q:6 (18 points) Find Fourier integral representation of

$$f(x) = \begin{cases} 0, & x < 0 \\ \sin 2x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

Q:7 (18 points) Use appropriate Fourier transform to solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \ y > 0,$$

subject to the conditions

$$\begin{aligned} u(0, y) &= e^{-y}, \quad u(\pi, y) = 0, \quad y > 0, \\ \left.\frac{\partial u}{\partial y}\right|_{y=0} &= 0, \qquad 0 < x < \pi. \end{aligned}$$