King Fahdfdsz University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Major Exam 2
The Second Semester of 2015-2016 (152)
Time Allowed: 120 Minutes

Name: _____ ID#: _____

Instructor: _____ Sec #: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		15
2		12
3		16
4		15
5		15
6		12
7		15
Total		100

Q:1 (15 points) Find the Laplace transform: $\mathcal{L}\{f(t)\}$, where $f(t) = \begin{cases} 2e^{2t}\sin(t), & 0 \le t < \frac{\pi}{2} \\ 3e^{-t}\cos(t), & t \ge \dfrac{\pi}{2} \end{cases}$.

Hint: Use Unit step function

Q:2 (12 points) Solve the integral equation using Laplace transform

$$f(t) = 2 + 3t + \frac{1}{6}\int_0^t (t-\tau)^3 f(\tau)d\tau.$$

Q:3 (8+8 points) Find the following:

(a) $\mathcal{L}\{te^{-2t}\cosh 3t\}$,

(b) $\mathcal{L}^{-1}\left\{\dfrac{(s+3)e^{-\pi s}}{s^2+4s+5}\right\}$,

Q:4 (15 points) Solve the boundary value problem using Laplace transform

$y'' + 6y' + 13y = \delta(t - \pi)$ with $y(0) = 1$, $y(\frac{\pi}{4}) = 1$.

Q:5 (10+5 points) Find the Fourier series of the function $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 \leq x < \pi \end{cases}$.

Use the Fourier series to show that $\frac{1}{2} = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots$

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Q:6 (12 points) Let $f(x) = \begin{cases} 0 & 0 < x < \dfrac{\pi}{2} \\ \pi - x & \dfrac{\pi}{2} \leq x < \pi \end{cases}$.

Find Half-Range Fourier series of $f(x)$.

Q:7 (15 points) Find eigenvalues and eigenfunctions of the boundary value problem

$$y'' + 2y' + \lambda y = 0 \text{ with } y(0) = 0 \text{ and } y(\pi) = 0.$$