King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Major Exam 1
The Second Semester of 2015-2016 (152)

Time Allowed: 120 Minutes

Name: ______________________ ID#: ____________

Instructor: ____________________ Sec #: ______ Serial #: ______

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		12
2		14
3		14
4		15
5		15
6		15
7		15
Total		100

Q:1 (6+6 points) Given the position vector $\vec{r}(t) = 2\sqrt{2}\, t\ \hat{\imath} + e^{2t}\hat{\jmath} + e^{-2t}\hat{k}$ of a curve C:

(a) Find $\dfrac{d}{dt}\left[\vec{r}(t) \times \vec{r}'(t)\right]$ at $t = 0$.

(b) Find the length of the curve for $0 \le t \le 2$.

Q:2 (8+6 points) Let $f(x, y, z) = xy - 3y^2z + x^2z$.

(a) Find the directional derivative of $f(x, y, z)$ at $(1, -1, 0)$ in the direction of $\hat{\imath} + 2\,\hat{\jmath} + 3\,\hat{k}$.

(b) Find the direction in which the function $f(x, y, z)$ decreases most rapidly at $(1, 1, 1)$ and the value of minimum rate of change at this point.

Q:3 (6+8 points) Let $g(x,y,z) = xyz$ and $\vec{F} = y\,\hat{i} - x\,\hat{j} + z;\hat{k}$. Calculate

(a) $\nabla \cdot (g\vec{F})$ at $(0,1,1)$

(b) $\nabla \times (g\vec{F})$ at $(1,1,1)$.

Q:4 (15 points) Find work done by the force $\vec{F} = (y + yz\cos x)\,\hat{i} + (x + z\sin x)\,\hat{j} + y\sin x\,\hat{k}$ along the curve $\vec{r}(t) = 2t\,\hat{i} + (1 + \cos t)\,\hat{j} + 4\sin t\,\hat{k}$ for $0 \le t \le \dfrac{\pi}{2}$.

Q:5 (15 points) Use Green's theorem to evaluate the line integral

$\oint_C (-16y + \sin x^2)dx + (4e^y + 3x^2)dy$, where C is the positively oriented boundary of the region bounded by the graphs of $y = x$, $y = -x$, and $x^2 + y^2 = 4$ with $y \geq 0$.

Q:6 (15 points) Use Stokes' theorem to evaluate the integral $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = y^3\hat{\imath} - x^3\hat{\jmath} + z^3\hat{k}$ and C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $x + y + z = 1$

Q:7 (15 points) Use divergence theorem to evaluate $\iint\limits_S (\vec{F}.\hat{n})\, dS$

where $\vec{F} = z^2 \sin y\ \hat{i} + 5x^2 z\ \hat{j} + 3z^2\ \hat{k}$ and D the region bounded by the surface S given by

$z = \sqrt{9 - x^2 - y^2}$, $x^2 + y^2 = 4$, $z = 0$.