King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

Math 280: Introduction to Linear Algebra

Final Exam, Spring Semester 152 (180 minutes)

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Remark: Show full details of your proofs and solutions.

Q1. (10 points) Find the dimension of the subspace S of $\mathbb{R}^3$ spanned by $\{(2,1,3)^T, (3,-1,4)^T, (2,6,4)^T\}$.

Q2. (10 points) Show the eigenvalues of a *Hermitian matrix* are all real.

Q3. (10 points) Let V be an inner product space. Show that any set $\{v_1, \cdots, v_n\}$ of non-zero orthogonal vectors is linearly independent.

Q4. (15 points) Find an orthonormal basis for $\mathbb{P}_3$ (the set of all polynomials of degree less than 3) with inner product defined by

$$< f, g >= \int_{-1}^{1} f(x)g(x)dx.$$

Q5. (10 points) Let $f(x)$ be a function whose curve passes through $(1,2)$, $(2,-1)$ and $(3,4)$. Use Lagrange's Interpolation to construct a second-degree polynomial that interpolates f at the given points.

Q6. (20 points) Consider the matrix

$$B = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 4 & 0 \\ -3 & 6 & 2 \end{bmatrix}$$

(a) Find the eigenvalues of B and the corresponding eigenspace.
(b) Show that B is diagonalizable.
(c) Find an invertible matrix P such that $P^{-1}BP$ is diagonal.
(d) Find P^{-1} from part "(c)" and calculate $P^{-1}BP$.

Q7. (15 points) Each year, the employees at a company are given the option of donating to a local charity. In general, 80% of the employees enrolled in the plan in any one year will choose to sign up again in the following year, and 30% of the unrolled will choose to enroll in the following year.

(a) Determine the transition matrix for this Markov process.

(b) Find the *steady state vector*.

(c) What percentage of the employees would you expect to find enrolled in the program on the long run.

Q8. (10 points) True of False.

(a) Every real matrix is similar to an upper triangular real matrix.

(b) Similar matrices have the same determinant.

(c) Any two vector spaces of the same dimension are isomorphic.

(d) Every finite dimensional inner product space has an orthonormal basis.

(e) Every one-to-one linear transformation is onto.

GOOD LUCK