**King Fahd University of Petroleum & Minerals**

**Department of Mathematics and Statistics**

**Math 280: Introduction to Linear Algebra**

**Second Exam, Spring Semester 152 (120 minutes)**

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**Q1. (10 points)** Show that $\{e^x, e^{-x}, e^{2x}\}$ is linearly independent in $C[0,1]$.

**Q2. (10 points)** Let $A$ be an $m \times n$ matrix. Show that

$$\dim(\text{row space of } A) = \dim(\text{column space of } A).$$

**Q3. (25 points)** Consider $L : \mathbb{P}_3 \longrightarrow \mathbb{P}_3$ defined by

$$L(p(x)) = p(x) - p'(x).$$

(a) Show that $L$ is a linear operator.
(b) Find $\text{Ker}(L)$.
(c) Find $\dim(\text{Ker}(L))$.
(d) Find $\text{Im}(L)$.
(e) Find $\dim(\text{Im}(L))$.

**Q4. (20 points)** Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 4 \\ -2 & -4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

(a) Find a basis for the row space of $A$.
(b) Find a basis for the column space of $A$.
(c) Find a basis for the Null space of $A$.

**Q5. (20 points)** Let $L : \mathbb{P}_3 \longrightarrow \mathbb{P}_3$ be the linear operator defined by

$$L(p(x)) = xp'(x) + p''(x).$$

(a) Find $[L]_E$, where $E = \{1, x, x^2\}$.
(b) Find $[L]_F$, where $F = \{1, x, 1 + x^2\}$.
(c) Find the matrix $S$ such that

$$[L]_F = S^{-1}[L]_E S.$$

(d) Calculate $L^n(p(x))$, where $p(x) = a_0 + a_1 x + a_2(1 + x^2)$.

**Q6. (15 points)** Prove or disprove:
(a) $\mathbb{R}^2$ is a vector space with the usual scalar multiplication and with the *new* addition

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + y_1, 0).$$

(b) $S = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + x_2 = 2\}$ is a subspace of $\mathbb{R}^3$.
(c) $\dim(Span\{1, \sin 2x, \sin x \cos x\}) = 2$.