**KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS**

MATH 232: EXAM 4, SEMESTER (152), APRIL 30, 2016

**06:30–09:30 pm**

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## PART A

**Exercise 1** (6 pts)**.** Consider the relation $R = \{(x, y) \in \mathbb{N} \times \mathbb{N} \mid x + 4y \text{ is odd}\}$.

(1) Define the relation $R^{-1}$.
(2) Is $R = R^{-1}$?

**Exercise 2** (6 pts)**.** Let $R$ be an equivalence relation on a set $A$ and $B \subseteq A$. Set $S = R \cap (B \times B)$.

(1) Show that $S$ is an equivalence relation on $B$.
(2) Show that for all $x \in B$, $[x]_S = [x]_R \cap B$.

**Exercise 3** (10 pts)**.** Let $a, b \in \mathbb{Z}$. Show that the relation $R$ defined on $\mathbb{Z}$ by:

$$aRb \qquad \text{iff } a^2 + b^2 \qquad \text{is even}$$

is an equivalence relation and determine its distinct equivalence classes.

**Exercise 4** (10 pts)**.** Give the addition and multiplication tables on $\mathbb{Z}_9$.

Find all integers $x$ such that $6x \equiv 3 \pmod 9$.

**Exercise 5** (8 pts)**.** Let $g : \mathbb{Z} \longrightarrow \mathbb{Z}$ be the function defined by: $g(x) = 4x + 1$ .

(1) Determine $g(E)$, where $E$ is the set of all even integers. Is $11 \in g(E)$?

(2) Determine $g^{-1}(\mathbb{N})$, $g^{-1}(E)$ and $g^{-1}(O)$, where $O$ is the set of all odd integers.

**Exercise 6** (10 pts)**.** Let $f : A \longrightarrow B$ be a function.

($a$) Show that $f$ is injective if and only if $f^{-1}(f(C)) = C$, for all $C \subseteq A$.

($b$) Show that $f$ is surjective if and only if $f(f^{-1}(D)) = D$, for all $D \subseteq B$.

**Exercise 7** (10 pts)**.** Let $f : \mathbb{R} \setminus \{11\} \longrightarrow \mathbb{R} \setminus \{7\}$, defined by $f(x) = \dfrac{7x+1}{x-11}$. Show that $f$ is a bijection and find $f^{-1}$.

**PART B**

**Exercise 8** (10 pts)**.** Let $n \geq 2$ be an integer. Using the **PHP** (Pigeonhole Principle), prove that in a collection of $n+1$ distinct integers, there are distinct integers $x$ and $y$ such that $x - y$ is a multiple of $n$.

**Exercise 9** (10 pts)**.** Find the number of positive integers $x$ between 1 and $10^4$ such that $7 \nmid x$ and $13 \nmid x$.

**Exercise 10** (10 pts)**.** Let $A = \{1, 2, 3\}$. Find the distinct equivalence classes of the equivalence relation $\sim$ defined on $S = P(A)$ by:

$$E \sim F \quad \text{iff} \quad \mid E \mid = \mid F \mid .$$

**Exercise 11** (10 pts)**.**

(1) Give an explicit injection from $\mathbb{N} \times \mathbb{N}$ into $\mathbb{N}$.
(2) For any integer $k \geq 3$ give an explicit injection from $\mathbb{N} \times \mathbb{N} \times \mathbb{N} \ldots \times \mathbb{N}$ ($k$ times) into $\mathbb{N}$.
(3) Deduce that $\mathbb{N} \times \mathbb{N} \times \mathbb{N} \ldots \times \mathbb{N}$ and $\mathbb{N}$ have the same cardinality.

**Exercise 12** (10 pts)**.** Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{a, b, c, d, e\}$.

(1) Find all the equivalence relations on $A$ with exactly 5 distinct equivalence classes.
(2) Show that if $f : A \longrightarrow B$ is an onto map then $\{f^{-1}(\{b\}) \mid b \in B\}$ is a partition of $A$.
(3) How many onto maps are there from $A$ to $B$?

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**Exercise 13** (10 pts)**.** Let $a < b$ be real numbers and $a < c < b$. Show that $]a, b[\setminus\{c\}$ and $\mathbb{R}$ have the same cardinality.