KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 232: EXAM III, SEMESTER (152), MARCH 22, 2016

Name : ....

ID : .....

Allowed Time : 1H 30mn

Exercise	**Points**
1	: **8**
2	: **15**
3	: **7**
4	: **10**
5	: **10**
6	: **10**
Total	: **60**

Exercise 1. Let $a, b \in \mathbb{R} \setminus \{0\}$. Show that the statement

$$\text{If} \quad x, y > 0 \quad \text{then} \quad \frac{a^2}{2b^2}x^2 + \frac{b^2}{2a^2}y^2 > xy$$

is false.

Exercise 2.

(1) Sow that if n is an integer which is not a perfect square int, then $\sqrt{n} \notin \mathbb{Q}$.

(2) Sow that $\sqrt{5}+\sqrt{7} \notin \mathbb{Q}$.

(3) Sow that $\left\{a+b\sqrt{5} : a, b \in \mathbb{Q}\right\} \cap \left\{p+q\sqrt{7} : p, q \in \mathbb{Q}\right\} = \mathbb{Q}$.

Exercise 3. Disprove the statement: There exists an integer n such that n^2+3n+3 is even.

Exercise 4. Use Mathematical induction to show that, for every positive integer n,

$$3 \times 5^{2n+1} + 2^{3n+1}$$

is a multiple of 17.

Exercise 5. Let $0 \leq a \leq 1$. Use Mathematical induction to show that, for each positive integer n,

$$(1-a)^n \geq 1 - na.$$

Exercise 6. Let (a_n) be the sequence defined recursively by.

$$a_1 = 4, a_2 = 9 \text{ and } a_{n+1} = 2a_n - a_{n-1} + 2, \text{ for } n \geq 3.$$

Find a formula for a_n.