KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 232: EXAM II, SEMESTER (152), MARCH 07, 2016

Name : ....

ID : .....

Allowed Time : 1H 30mn

Exercise	**Points**
1	: **4**
2	: **11**
3	: **9**
4	: **9**
5	: **9**
6	: **9**
7	: **9**
Total	: **60**

Exercise 1. Let X be a set and

$$S = \{A \in X \mid A \text{ is a set and } A \notin A\}.$$

Show that $S \notin X$.

Exercise 2. Let A, B be two sets.

(1) Sow that the following properties hold:
   (a) $A = (A - B) \cup (A \cap B)$.
   (b) $(A \cup B) = A \cup (B - A)$.
   (c) $(A \Delta B) = (A \cup B) - (A \cap B)$.

(2) Use the previous results and the additive rule to show that the following properties hold:
   (a) $| A - B | = | A | - | A \cap B |$.
   (b) $| A \cup B | = | A | + | B | - | A \cap B |$.
   (c) $| A \Delta B | = | A | + | B | - 2 | A \cap B |$.

Exercise 3. Let $A = \{1, 2, 3, 4, 5\}$. Find all partitions of A into 4 blocks.

Exercise 4. Let $x \in \mathbb{R}$. Show that the following two conditional statements hold:

(1) If $x > 2015$, then $x^{2016} + x^2 - 4x + 5 > 0$.

(2) If $x^{2016} + x^2 - 6x + 10 = 0$, then $x^{2018} > x^{2017}$.

Exercise 5. Let $x \in \mathbb{Z}$. Show that if $x^2 + 2x - 5$ is odd, then $3x + 4$ is even.

Exercise 6. Let x be an integer. Show that if $4 \nmid (x^2 - 1)$, then $4 \mid x$ or $4 \mid (x - 2)$.

Exercise 7. Let x be an integer. Show that if $x \not\equiv 2 \pmod 6$, then $x(x^2 - 1) \equiv 0 \pmod 6$.