MATH 202.5 (Term 152)

Quiz 5 (Sects. 6.2 & 6.3) Duration: 30min

Name: ID number:

1.)(5pts)Find 2 powers series solutions of the DE: $(x-4)y''-y'-2y=0$.
2.)(2pts)Find the indicial roots of the DE $x^2y''+3x(x^4-1)y'-(1-x^3)y=0$ at $x=0$.
3.)(3pts)Find a relation of recurrence satisfied by c_n, where $y=\sum_{n=0}^{\infty}c_nx^{n+\frac{1}{2}}$ is solution of the DE $2xy''+y'+2y=0$.

1) $y=\sum_{n=0}^{\infty}c_nx^n$

$(x-4)\sum_{n=2}^{\infty}c_nn(n-1)x^{n-2}-\sum_{n=1}^{\infty}c_nnx^{n-1}-2\sum_{n=0}^{\infty}c_nx^n=0$

$-8c_2-c_1-2c_0+\sum_{k=1}^{\infty}\Big[c_{k+1}(k+1)(k+2)$
$-4c_{k+2}(k+1)(k+2)-2c_k\Big]x^k=0$

$\begin{cases} c_2=-\dfrac{c_1+2c_0}{8} \\ c_{k+2}=\dfrac{(k+1)(k\ \ )c_{k+1}-2c_k}{4(k+1)(k+2)}, & k=1,2,\ldots \end{cases}$

Case 1: $\boxed{c_0=0,\ c_1\neq 0}$

$c_2=-\dfrac{c_1}{8}$

$c_3=-\dfrac{c_1}{12}$, $c_4=0$

Case 2 $\boxed{c_0\neq 0,\ c_1=0}$

$c_2=-\dfrac{c_0}{4}$, $c_3=0$

$c_4=\dfrac{c_0}{96}$

thus,

$y=c_1\Big(\underbrace{x-\frac{x^2}{8}-\frac{1}{12}x^3+\cdots}_{y_1}\Big)$

$y=c_0\Big(\underbrace{1-\frac{x^2}{4}+\frac{x^4}{96}+\cdots}_{y_2}\Big)$

$\boxed{y=c_1y_1+c_2y_2 \text{ is the general solution}}$

2.) $p(x)=3(x^4-1)$; $q(x)=-1(1-x^3)$

$r(r-1)-3r-1=0$; $r^2-4r-1=0$

$r_1=2-\sqrt{5}$; $r_2=2+\sqrt{5}$

3.) $y=\sum_{n=0}^{\infty}c_nx^{n+\frac{1}{2}}$, $y'=\sum_{n=0}^{\infty}c_n(n+\tfrac{1}{2})x^{n-\frac{1}{2}}$

$y''=\sum_{n=0}^{\infty}c_n(n+\tfrac{1}{2})(n-\tfrac{1}{2})x^{n-\frac{3}{2}}$

$\Rightarrow \sum_{n=1}^{\infty}2c_n(n+\tfrac{1}{2})(n-\tfrac{1}{2})x^{n-1}+\sum_{n=1}c_n(n+\tfrac{1}{2})x^{n-1}$

$+\sum_{n=0}2c_nx^n=0$

$\sum_{k=0}^{\infty}2c_{k+1}(k+\tfrac{3}{2})(k+\tfrac{1}{2})x^k+\sum_{k=0}^{\infty}c_{k+1}(k+\tfrac{3}{2})x^k$

$+\sum_{k=0}^{\infty}2c_kx^k=0$

$\Rightarrow$

$\boxed{c_{k+1}=\dfrac{-2c_k}{(k+1)(2k+3)},\ k=0,1,2}$

MATH 202.10 (Term 152)

Quiz 5 (Sects. 6.2 & 6.3) Duration: 30min

Name: ID number:

1.)(5pts)Find 2 powers series solutions of the DE: $(x-3)y''-xy'-y=0$.
2.)(2pts)Find the indicial roots of the DE $x^2y''-2x(1-x^5)y'-(1-x^6)y=0$ at $x=0$.
3.)(3pts)Find a relation of recurrence satisfied by c_n, where $y=\sum_{n=0}^{\infty} c_n x^{n-\frac{1}{2}}$ is solution of the DE $2xy''+3y'+2y=0$.

1) $y=\sum_{n=0}^{\infty} c_n x^n$

$(x-3)\sum_{n=2}^{\infty} c_n n(n-1)x^{n-2} - x\sum_{n=1}^{\infty} c_n n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$

$-6c_2 - c_0 + \sum_{k=1}^{\infty}\left[c_{k+1}(k+1)k - 3c_{k+2}(k+1)(k+2) - c_k(1+k)\right]x^k = 0$

$$\begin{cases} c_2 = -\frac{c_0}{6} \\ c_{k+2} = \frac{k c_{k+1} - c_k}{3(k+2)}, \quad k=1,2,\dots \end{cases}$$

case 1: $\boxed{c_0 = 0,\ c_1 \neq 0}$

$c_2 = 0$

$c_3 = -\frac{c_1}{9}$, $c_4 = -\frac{c_1}{54}$

$\Rightarrow y = c_1\underbrace{\left(x - \frac{x^3}{9} - \frac{x^4}{54} + \cdots\right)}_{y_1}$

case 2 $\boxed{c_0 \neq 0,\ c_1 = 0}$

$c_3 = \frac{-c_0}{54}$

$y = c_0\underbrace{\left(1 - \frac{x^2}{6} - \frac{x^3}{54} + \cdots\right)}_{y_2}$

$\boxed{y = c_1 y_1 + c_2 y_2 \text{ is the general solution}}$

2) $p(x) = -2(1-x^5)$; $q(x) = -(1-x^6)$

$r(r-1) - 2r - 1 = 0$, $r^2 - 3r - 1 = 0$

$r_1 = \frac{3-\sqrt{13}}{2}$, $r_2 = \frac{3+\sqrt{13}}{2}$

3) $y = \sum_{n=0}^{\infty} c_n x^{n-\frac{1}{2}}$, $y' = \sum_{n=0}^{\infty} c_n (n-\frac{1}{2}) x^{n-\frac{3}{2}}$

$y'' = \sum_{n=0}^{\infty} c_n (n-\frac{1}{2})(n-\frac{3}{2}) x^{n-\frac{5}{2}}$

$\Rightarrow \sum_{n=1}^{\infty} 2c_n(n-\frac{1}{2})(n-\frac{3}{2})x^{n-1} + \sum_{n=1}^{\infty} 3c_n(n-\frac{1}{2})x^{n-1} + \sum_{n=0}^{\infty} 2c_n x^n = 0$

$\sum_{k=0}^{\infty} 2c_{k+1}(k+\frac{1}{2})(k-\frac{1}{2})x^k + \sum_{k=0}^{\infty} 3c_{k+1}(k+\frac{1}{2})x^k + \sum_{k=0}^{\infty} 2c_k x^k = 0$

$$\boxed{c_{k+1} = \frac{-2c_k}{(k+1)(2k+1)}, \quad k=0,1,2.}$$