MATH 202.5 (Term 152)

Quiz 3 (Sects. 4.2, 4.3 & 4.5) Duration: 20min

Name: **ID number:**

1.)(3pts)Use reduction of order (only) to find a second solution y_2 of the DE: $4x^2y'' + 5y = 0$, **giving that** $y_1 = x^{1/2}\cos\ln x$ **is a solution on** $(0, \infty)$.

2.)(3pts)Solve the DE: $2y''' - y'' - 2y' + y = 0$.

3.)(4pts) Find an operator of lowest order that annihilates $f(x) = x^2e^{-x}\sin 2x\cos x$

Hint: $\sin(a+b) = \sin a\cos b + \cos a\sin b$.

1) $y'' + \frac{5}{4x^2}y = 0$, $P(x) = 0$

$y_2 = y_1\int\frac{e^{-\int P(x)dx}}{y_1^2}dx$

$= y_1\int\frac{1}{x\cos^2(\ln x)}dx$

Let $u = \ln x$, $du = \frac{dx}{x}$

$\Rightarrow y_2 = y_1\int\frac{1}{\cos^2 u}du$

$= y_1\tan u = y_1\frac{\sin(\ln x)}{\cos(\ln x)}$

$\boxed{y_2 = x^{1/2}\sin\ln x}$

2) Its auxiliary equation is

$2m^3 - m^2 - 2m + 1 = 0$

$m = 1$ is a root.

$$\begin{array}{l|l} 2m^3 - m^2 - 2m + 1 & m-1 \\ \hline -(2m^3 - 2m^2) & 2m^2 + m - 1 \\ \quad m^2 - 2m & \\ \quad -(m^2 - m) & \\ \qquad -m + 1 & \\ \qquad -(-m + 1) & \\ \qquad\quad 0 & \end{array}$$

$\Delta = 1 + 8 = 9$

$m_1 = \frac{-1-3}{4} = -1$, $m_2 = \frac{-1+3}{4} = \frac{1}{2}$

$\Rightarrow \boxed{y = c_1e^{-x} + c_2e^{x} + c_3e^{x/2},\ x \in (-\infty, \infty)}$

3.) $\sin a\cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$

$\Rightarrow \sin 2x\cos x = \frac{1}{2}[\sin 3x + \sin x]$

$\Rightarrow f(x) = \frac{x^2e^{-x}}{2}\sin 3x + \frac{x^2e^{-x}}{2}\sin x$

$\downarrow \qquad\qquad \downarrow$

$(D^2 + 2D + 10)^3 \quad (D^2 + 2D + 2)^3$

$\Rightarrow \boxed{(D^2 + 2D + 10)^3(D^2 + 2D + 2)^3[f(x)] = 0}$

MATH 202.10 (Term 152)

Quiz 3 (Sects. 4.2, 4.3 & 4.5) Duration: 20min

Name: ID number:

1.)(3pts)Use reduction of order (only) to find a second solution y_2 of the DE: $x^2y'' - 2y = 0$, **giving that** $y_1 = x^2$ **is a solution on** $(0, \infty)$.

2.)(3pts)Solve the DE: $3y''' - 2y'' - 3y' + 2y = 0$.

3.)(4pts) Find an operator of lowest order that annihilates $f(x) = x^3 e^x \cos 2x \cos x$

Hint: $\cos(a+b) = \cos a \cos b - \sin a \sin b$.

1.) $y'' - \frac{2}{x^2} y = 0$, $P(x) = 0$

$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$

$= y_1 \int \frac{1}{x^4}\,dx$

$= y_1 \left(\frac{1}{-3x^3}\right) = \frac{x^2}{-3x^3}$

$\Rightarrow \boxed{y_2 = \frac{1}{x}}$

2.) The auxiliary equation of the DE is

$3m^3 - 2m^2 - 3m + 2 = 0$

$m = 1$ is a root.

$$\begin{array}{l|l} 3m^3 - 2m^2 - 3m + 2 & m-1 \\ -\ 3m^3 - 3m^2 & 3m^2 + m - 2 \\ \hline \quad m^2 - 3m & \\ -\ m^2 - m & \\ \hline \quad -2m + 2 & \\ \quad -2m + 2 & \\ \hline \quad 0 & \end{array}$$

$\Delta = 1 + 24 = 25$

$m_1 = \frac{-1-5}{6} = -1$, $m_2 = \frac{-1+5}{6} = \frac{2}{3}$

$\Rightarrow \boxed{y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{\frac{2x}{3}},\ x \in (-\infty, \infty)}$

3.) $\cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$

$\Rightarrow \cos 2x \cos x = \frac{1}{2}[\cos 3x + \cos x]$

$\Rightarrow f(x) = \frac{x^3 e^x \cos 3x}{2} + \frac{x^3 e^x \cos x}{2}$

$\downarrow \qquad\qquad \downarrow$

$(D^2 - 2D + 10)^4 \qquad (D^2 - 2D + 2)^4$

$\Rightarrow \boxed{(D^2 - 2D + 10)^4 (D^2 - 2D + 2)^4 [f(x)] = 0}$