MATH 202.5 (Term 152)

Quiz 2 (Sects. 2.4 & 2.5) Duration: 20min

Name: ID number:

1.)(5pts)Solve the DE: $ye^{-xy} + x\cos 2x + xe^{-xy}\frac{dy}{dx} = 0$.

2.)(5pts)Solve the DE: $(x+1)^2\frac{dy}{dx} - y = y^2$.

1) $\underbrace{(ye^{-xy} + x\cos 2x)}_{M}\,dx + \underbrace{xe^{-xy}}_{N}\,dy = 0$

$M_y = e^{-xy} - xy\,e^{-xy}$

$N_x = e^{-xy} - xy\,e^{-xy}$

$\Big\}\ M_y = N_x \Rightarrow$ DE is exact.

$$\begin{cases} \frac{\partial f}{\partial x} = ye^{-xy} + x\cos 2x & (1) \\ \frac{\partial f}{\partial y} = xe^{-xy} & (2) \end{cases}$$

We integrate (2)

$f(x,y) = \int xe^{-xy}\,dy$

$= -e^{-xy} + g(x)$

We substitute into (1)

$ye^{-xy} + g'(x) = ye^{-xy} + x\cos 2x$

$g'(x) = x\cos 2x$

$g(x) = \int x\cos 2x\,dx$

$u = x \to u' = 1$

$v' = \cos 2x \to v = \frac{\sin 2x}{2}$

$\Rightarrow g(x) = \frac{x\sin 2x}{2} + \frac{\cos 2x}{4}$

Thus, $\boxed{-e^{-xy} + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} = C}$

or $\boxed{y = \frac{-\ln\left|C + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right|}{x}, x \in I}$

2.) $y = 0$ is a trivial solution.

Assume $y \neq 0$: $\frac{dy}{dx} - \frac{1}{(x+1)^2}y = \frac{y^2}{(x+1)^2}$, $x \neq -1$

This is Bernoulli's DE

let $u = y^{1-2} = y^{-1}$

$\Rightarrow y = u^{-1}$, $\frac{dy}{dx} = -u^{-2}\frac{du}{dx}$

$\Rightarrow -u^{-2}\frac{du}{dx} - \frac{u^{-1}}{(x+1)^2} = \frac{u^{-2}}{(x+1)^2}$, $x \neq -1$

$\frac{du}{dx} + \frac{u}{(x+1)^2} = -\frac{1}{(x+1)^2}$, $x \neq -1$

$e^{\int\frac{dx}{(x+1)^2}} = e^{-\frac{1}{x+1}}$ is an integrating factor

$\Rightarrow \frac{d}{dx}\left(u e^{-\frac{1}{x+1}}\right) = -\frac{e^{-\frac{1}{x+1}}}{(x+1)^2}$

$u e^{-\frac{1}{x+1}} = -\int \frac{e^{-\frac{1}{x+1}}}{(x+1)^2}\,dx$

$v = -\frac{1}{x+1}$, $dv = \frac{1}{(x+1)^2}dx$

$= -\int e^v\,dv = -e^v + C$

Thus,

$u e^{-\frac{1}{x+1}} = -e^{-\frac{1}{x+1}} + C$

$u = -1 + Ce^{\frac{1}{x+1}}$

$\boxed{y = \frac{1}{-1 + Ce^{\frac{1}{x+1}}}, \quad x \in (-1, \infty)}$

MATH 202.10 (Term 152)

Quiz 2 (Sects. 2.4 & 2.5) Duration: 20min

Name: ID number:

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**1.)(5pts)Solve the DE:** $3x^2y - x\sin x - y^2e^x = (2ye^x - x^3)\frac{dy}{dx}$.

**2.)(5pts)Solve the DE:** $(e^{2x}+1)\frac{dy}{dx} + 2(e^{2x}+1)y = y^{1/2}$.

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1.) $\underbrace{(3x^2y - x\sin x - y^2e^x)}_{M}\,dx - \underbrace{(2ye^x - x^3)}_{N}\,dy = 0$

$\left.\begin{array}{l} M_y = 3x^2 - 2ye^x \\ N_x = -2ye^x + 3x^2 \end{array}\right\}$ $M_y = N_x$, DE is exact

$$\begin{cases} \dfrac{\partial f}{\partial x} = 3x^2y - x\sin x - y^2e^x & (1) \\ \dfrac{\partial f}{\partial y} = x^3 - 2ye^x & (2) \end{cases}$$

We integrate (2) $\Rightarrow$

$f(x,y) = yx^3 - y^2e^x + g(x)$

We substitute into (1)

$3x^2y - y^2e^x + g'(x) = 3x^2y - x\sin x - y^2e^x$

$\Rightarrow g'(x) = -x\sin x$

$g(x) = -\int x\sin x\,dx$

$u = x \to u' = 1$

$v' = -\sin x \to v = \cos x$

$g(x) = x\cos x - \sin x$

Thus,

$$\boxed{yx^3 - y^2e^x + x\cos x - \sin x = C}$$

$x \in I$.

$y = y(x)$ is an implicit solution

2.) $y = 0$ is a trivial solution.

Now, assume $y \neq 0$.

$\frac{dy}{dx} + 2y = \frac{y^{1/2}}{e^{2x}+1}$

This is Bernoulli's DE

let $u = y^{1-\frac{1}{2}} = y^{1/2}$

$y = u^2$, $\frac{dy}{dx} = 2u\frac{du}{dx}$

$\Rightarrow 2u\frac{du}{dx} + 2u^2 = \frac{u}{e^{2x}+1}$

$u \neq 0 \Rightarrow \frac{du}{dx} + u = \frac{1}{2}\frac{1}{e^{2x}+1}$

$e^{\int dx} = e^x$ is an integrating factor

$\frac{d}{dx}(ue^x) = \frac{e^x}{2(e^{2x}+1)}$

$ue^x = \frac{1}{2}\int \frac{e^x}{e^{2x}+1}\,dx$

$v = e^x$, $dv = e^x\,dx$

$= \frac{1}{2}\int \frac{dv}{v^2+1} = \frac{1}{2}\tan^{-1}v + C$

$\Rightarrow ue^x = \frac{1}{2}\tan^{-1}(e^x) + C$

$e^x y^{\frac{1}{2}} = \frac{1}{2}\tan^{-1}(e^x) + C$

$$\boxed{y = \left(\frac{\tan^{-1}(e^x) + C}{2e^x}\right)^2, \quad x \in (-\infty, \infty)}$$