

Name: _____

ID number: _____

1.) (5pts) Solve the IVP: $\begin{cases} \frac{dy}{dx} = y(y+1)\sin^2 x \\ y(0) = 1. \end{cases}$

2.) (5pts) Solve the DE: $x \frac{dy}{dx} - y = \frac{x^2}{x^2+1}$.

1.) The trivial solutions are

$$y=0 \text{ and } y=-1$$

But, both solutions are rejected because they don't satisfy $y(0)=2$.

Now, we look for non trivial solutions.

We assume $y \neq 0, -1$

The DE is separable.

$$\int \frac{dy}{y(y+1)} = \int \sin^2 x \, dx$$

$$\int \left(\frac{1}{y} - \frac{1}{y+1} \right) dy = \int \frac{1 - \cos 2x}{2} \, dx$$

$$\ln \left| \frac{y}{y+1} \right| = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$y(0)=1 \Rightarrow C = \ln\left(\frac{1}{2}\right)$$

$$\frac{y}{y+1} = \frac{1}{2} e^{\frac{x}{2} - \frac{\sin 2x}{4}}$$

or,
$$y = \frac{\frac{1}{2} e^{\frac{x}{2} - \frac{\sin 2x}{4}}}{1 - \frac{1}{2} e^{\frac{x}{2} - \frac{\sin 2x}{4}}}, x \in I$$

2.) We can't find trivial solutions

This is a linear DE

$$\frac{dy}{dx} - \frac{y}{x} = \frac{x}{x^2+1}$$

$$e^{-\int \frac{dx}{x}} = e^{-\ln|x|} = \frac{1}{x}, x > 0, \text{ is}$$

an integrating factor -

Thus, we have

$$\frac{d}{dx} \left(\frac{y}{x} \right) = \frac{1}{x^2+1}$$

$$\frac{y}{x} = \int \frac{dx}{x^2+1}$$

$$= \tan^{-1} x + C$$

$$y = x (\tan^{-1} x + C), x \in (0, \infty)$$

Name: _____

ID number: _____

1.) (5pts) Solve the IVP: $\begin{cases} \frac{dy}{dx} = y(y-1)\cos^2 x \\ y(0) = 2. \end{cases}$

2.) (5pts) Solve the DE: $x\frac{dy}{dx} + y = \frac{x}{x^2+1}$.

1.) Trivial solutions,

$$y=c, \quad c(c-1)=0$$

$$y=0 \text{ and } y=1$$

Both solutions are rejected because they don't satisfy $y(0)=2$.

Now, we look for non trivial solutions.

Assume $y \neq 0, 1$

The DE is separable. Thus,

$$\int \frac{dy}{y(y-1)} = \int \cos^2 x \, dx$$

$$\int \left(\frac{1}{y-1} - \frac{1}{y} \right) dy = \int \frac{1 + \cos 2x}{2} dx$$

$$\ln \left| \frac{y-1}{y} \right| = \frac{2x + \sin 2x}{4} + C$$

$$y(0)=2 \Rightarrow \ln\left(\frac{1}{2}\right) = C$$

$$\Rightarrow \frac{y-1}{y} = \frac{1}{2} e^{\frac{2x + \sin 2x}{4}}$$

$$\text{or } \boxed{y = \frac{1}{1 - \frac{1}{2} e^{\frac{2x + \sin 2x}{4}}} \quad | \quad x \in I}$$

2.) No trivial solution can be found.

This is a linear DE

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2+1}, \quad x \neq 0$$

$$e^{\int \frac{dx}{x}} = e^{\ln|x|} = x, \quad x > 0, \text{ is an integrating factor.}$$

Thus, we have

$$\frac{d}{dx}(yx) = \frac{x}{x^2+1}$$

$$yx = \int \frac{x}{x^2+1} dx$$

$$= \frac{1}{2} \ln(x^2+1) + C$$

$$\boxed{y = \frac{\ln(x^2+1)}{2x} + C, \quad x \in (0, \infty)}$$