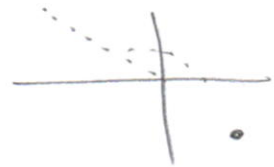
KFUPM MATH201 Quiz#2 Sec. 1

Name Sr.#

**Q1.** Find the polar coordinates, $0 \le \theta \le 2\pi$ and $r \le 0$ of the point $(\sqrt{3}, -1)$

$r^2 = x^2 + y^2 \rightarrow r^2 = 4 \rightarrow r = \pm 2 \rightarrow \boxed{r = -2}$

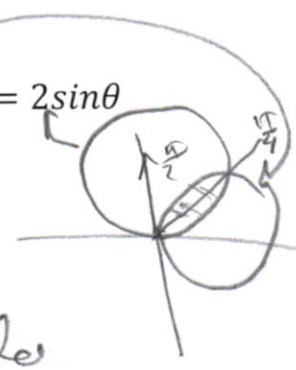
$\tan\theta = \frac{y}{x} = -\frac{1}{\sqrt{3}} \qquad \theta' = \frac{\pi}{6}$



$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\boxed{\left(-2, \frac{5\pi}{6}\right)}$

**Q2.** Find the area of the region shared by the circles $r = 2\cos\theta, \quad r = 2\sin\theta$



Intersection $2\cos\theta = 2\sin\theta \rightarrow \theta = \frac{\pi}{4}$

$$A = \int_0^{\frac{\pi}{4}} \frac{1}{2}[2\sin\theta]^2 d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}[2\cos\theta]^2 d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \frac{1-\cos 2\theta}{2} d\theta + 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2} d\theta$$

$$= \theta - \frac{\sin 2\theta}{2}\Big]_0^{\frac{\pi}{4}} + \theta + \frac{\sin 2\theta}{2}\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left(\frac{\pi}{4} - \frac{1}{2}\right) - (0) + \left(\frac{\pi}{2} + 0\right) - \left[\frac{\pi}{4} + \frac{1}{2}\right]$$

$$= \frac{\pi}{2} - 1$$