| Name: KEY | ID: KEY | Sec: 8(9:00-9:50) 9(10:00-10:50) |
|---|---|---|

MATH-102 Term-152 CQ13 Form B

**(show all your work and circle one letter to get a full mark or you will get zero)**

1) The interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-2)^n}{n}(x-3)^n$ is

10 points

$R=\lim_{n\to\infty}\left|\frac{(-2)^{n+1}(x-3)^{n+1}}{n}\cdot\frac{(n+1)}{(-2)^n(x-3)^n}\right|=2|x-3|$

$2|x-3|<1 \Rightarrow |x-3|<1/2 \to (\frac{5}{2},7/2)$

$x=\frac{7}{2}\to\sum_{n=1}^{\infty}\frac{(-1)^n2^n}{n}(\frac{1}{2})^n=\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ conv alt. Harmonic

$x=\frac{5}{2}\to\sum_{n=1}^{\infty}\frac{(-1)^n2^n}{n}(-\frac{1}{2})^n=\sum_{n=1}^{\infty}\frac{(-1)^n2^n}{n}\frac{(-1)^n}{2^n}=\sum_{n=1}^{\infty}\frac{1}{n}$

harmonic divg

$(\frac{5}{2},7/2]$

(a) (5/4 , 7/4]
(b) [5/4 , 7/4]
(c) [5/2 , 7/2]
(d) (5/2 , 7/2)
(e) (5/2 , 7/2]
(f) None of the above

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**( TRUE or FALSE )** 20 points (4 each)

2) *If the power series* $f(x)=\sum_{n=0}^{\infty}c_n(x+\frac{3}{2})^n$ has radius of convergence R = 2 , then $(-\frac{7}{2},\frac{1}{2})=I$

center $=-3/2 \Rightarrow$ interval $(-\frac{3}{2}-2,-\frac{3}{2}+2)$

| | Statement | True or False | Reason |
|---|---|---|---|
| (a) | $\lim_{n\to\infty}\frac{c_n}{2^n}=0$ | ( T ) ( F ) | $x=-1\in I \Rightarrow \sum_{n=0}^{\infty}\frac{c_n}{2^n}$ conv $\Rightarrow \lim\frac{c_n}{2^n}=0$ |
| (b) | $\lim_{n\to\infty}(-1)^n\left(\frac{5}{2}\right)^n c_n\neq 0$ | ( T ) ( F ) | $x=-4\notin I \to \sum_{n=0}^{\infty}c_n(-1)^n(\frac{5}{2})^n$ divg |
| (c) | $\sum_{n=1}^{\infty}\frac{nc_n}{2^{n-1}}$ is convergent | ( T ) ( F ) | $f'(x)=\sum_{n=0}^{\infty}nc_n(x+\frac{3}{2})^{n-1}$ <br> $x=-1\in I\Rightarrow\sum_{n=0}^{\infty}\frac{nc_n}{2^{n-1}}$ conv |
| (d) | $\sum_{n=1}^{\infty}nc_n$ is convergent | ( T ) ( F ) | let $x=-1/2$ in $f'(x)$ <br> $\sum_{n=0}^{\infty}nc_n$ conv $-1/2\in I$ |
| (e) | $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}c_n}{n+1}$ is divergent | ( T ) ( F ) | $\int f(x)\,dx=C+\sum_{n=0}^{\infty}\frac{c_n(x+\frac{3}{2})^{n+1}}{n+1}$ |

$x=-1/2\in I \to C+\sum_{n=0}^{\infty}\frac{c_n}{n+1}$ conv

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