Name: KEY	ID: KEY	Sec: 8(9:00-9:50) 9(10:00-10:50)

MATH-102 Term-152 LCQ-power series **FORM A**

(show all your work and circle one letter to get a full mark or you will get zero)

1) The Taylor series for the function $f(x)=\sqrt{x}$ about $a=1$ is given by $f(x)=x^{1/2}$

(a) $1-\frac{1}{2}(x-1)+\frac{1}{4}(x-1)^2+\frac{3}{8}(x-1)^3+\cdots$

(b) $1+\frac{1}{2}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3+\cdots$

(c) $1+\frac{1}{2}(x-1)-\frac{1}{4}(x-1)^2+\frac{5}{8}(x-1)^3+\cdots$

(d) $1+\frac{1}{2}(x-1)+(x-1)^2+\frac{3}{8}(x-1)^3+\cdots$

(e) $1+(x-1)-\frac{1}{4}(x-1)^2+\frac{3}{8}(x-1)^3+\cdots$

(f) none of the above

$f'(x)=\frac{1}{2}x^{-1/2}$, $f''(x)=\frac{-1}{4}x^{-3/2}$

$f(1)=1$, $f'(1)=\frac{1}{2}$, $f''(1)=-\frac{1}{4}$

$C_2=\frac{-1/4}{2!}=-\frac{1}{8}$

2) Using the binomial series, we have, for $|x|<\frac{1}{2}$

$\sqrt{4+32x^3}=(4+32x^3)^{1/2}=(4)^{1/2}(1+8x^3)^{1/2}$

$=2(1+8x^3)^{1/2}$ $k=\frac{1}{2}$

(a) $1+\frac{1}{4}x^3+8x^6+32x^9+\cdots$

(b) $1+\frac{1}{4}x^3-8x^6+32x^9+\cdots$

(c) $2+8x^3+16x^6+64x^9+\cdots$

(d) $2-8x^3-16x^6-64x^9+\cdots$

(e) $2+8x^3-16x^6+64x^9+\cdots$

(f) none of the above

$(1+8x^3)^{1/2}=1+\frac{1}{2}(8x^3)+\frac{\frac{1}{2}(-\frac{1}{2})}{2!}(8x^3)^2+\cdots$

$=1+4x^3-8x^6+\cdots$

$\sqrt{4+32x^3}=2+8x^3-16x^6+\cdots$

3) The power series representation for the function $f(x)=\dfrac{9x^2}{2+6x^2}$ is $=\frac{9x^2}{2}\frac{1}{1+3x^2}$

(a) $\sum_{n=0}^{\infty}(-1)^n 3^n x^{2n}$

(b) $\sum_{n=0}^{\infty}(-1)^n 3^{n+2} x^{2n+2}$

(c) $\sum_{n=0}^{\infty}\frac{(-1)^n 3^{n+2} x^{2n+2}}{2}$

(d) $\sum_{n=0}^{\infty}(-1)^n 3^{n+1} x^{2n+2}$

(e) $\frac{9}{2}\sum_{n=1}^{\infty}(-1)^n 3^n x^{2n+2}$

(f) none of the above

$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$ $x\to -3x^2$

$\frac{1}{1+3x^2}=\sum_{n=0}^{\infty}(-1)^n 3^n x^{2n}$

multiply by $\frac{9x^2}{2}$

$\frac{9x^2}{2(1-x)}=\sum_{n=0}^{\infty}\frac{(-1)^n 3^{n+2}x^{2n+2}}{2}$

4) if the Maclaurin series of $e^x\sin x$ is $A+Bx+Cx^2+Dx^3+\cdots\cdots$ then C+D =

(a) 1/2

(b) 4/3

(c) 5/6

(d) 0

(e) 1

(f) none of the above

$e^x\sin x=(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\cdots)$
$*(x-\frac{1}{6}x^3+\frac{1}{120}x^5-\cdots)$

Coeff of $x^2\Rightarrow x^2 \Rightarrow C=1$

Coeff of $x^3\Rightarrow (1)(-\frac{1}{6})x^3+\frac{1}{2}x^2(x)$

$\Rightarrow D=-\frac{1}{6}+\frac{1}{2}=\frac{1}{3}$

$\Rightarrow C+D=1+\frac{1}{3}=4/3$

5) $\int\frac{1}{x^2}\cos(x^3)dx=$

(a) $c+\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n-1}}{(2n)!(6n-1)}$

(b) $c+\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n-2}}{(2n)!(6n-2)}$

(c) $c+\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n-1}}{(2n+1)!(6n-1)}$

(d) $c+\sum_{n=3}^{\infty}\frac{(-1)^n x^{6n-1}}{(2n)!(6n-1)}$

(e) $c+\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n-1}}{(2n)!(3n-1)}$

(f) none of the above

$\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$

$\cos x^3=\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n}}{(2n)!}$

$\frac{1}{x^2}\cos x^3=\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n-2}}{(2n)!}$

$\int\frac{1}{x^2}\cos x^3dx=c+\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\frac{x^{6n-1}}{(6n-1)}$

$=c+\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\frac{x^{6n-1}}{(6n-1)}$

6) Let $g(x)=x^3\tan^{-1}x$ and let $g''(x)=\sum_{n=0}^{\infty}c_n x^n$ then $c_{10}=$

(a) 149/81

(b) 153/17

(c) 132/9

(d) 232/17

(e) 137/89

(f) none of the above

$\tan^{-1}x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$

$g=x^3\tan^{-1}x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+4}}{2n+1}$

$g''(x)=\sum_{n=0}^{\infty}(-1)^n\frac{(2n+4)(2n+3)x^{2n+2}}{(2n+1)}$

$2n+2=10\Rightarrow n=4$

when $n=4\Rightarrow \frac{(-1)^4(12)(11)}{(9)}x^{10}=$

$=132/9\,x^{10}$

7) $\sum_{n=1}^{\infty} \frac{n}{2^{n-1}} =$ $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ diff

$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1}$ let $x=\frac{1}{2}$

$\frac{1}{(1-\frac{1}{2})^2} = \sum_{n=1}^{\infty} \frac{n}{2^{n-1}}$

$4 =$

(a) 9
(b) 1/7
(c) 13/9
(d) 22
(e) 4
(f) none of the above

8) Let $g(x) = (1+x^3)^{2/3}$ and let $g''(x) = \sum_{n=0}^{\infty} c_n x^n$ then $c_{10} =$

$k = 2/3$

the term containg x^{12} in $g(x)$ is

$\frac{(\frac{2}{3})(\frac{2}{3}-1)(\frac{2}{3}-2)(\frac{2}{3}-3)}{4!}(x^3)^4$

coeff of x^{10} in $g''(x)$ is

$\frac{(\frac{2}{3})(-\frac{1}{3})(-\frac{4}{3})(-\frac{7}{3})}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1} \times \cancel{12}\times 11 = \frac{-11\times 7\times 4}{81}$

(a) -308/81
(b) -77/27
(c) -44/9
(d) 0
(e) 2
(f) none of the above

9) The interval of convergence of the power series $\sum_{n=1}^{\infty}(\sqrt{n+1}-\sqrt{n})(x-5)^n$ is

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(\sqrt{n+2}-\sqrt{n+1})(x-5)^{n+1}}{(\sqrt{n+1}-\sqrt{n})(x-5)^n}\right|$

$= \left|\frac{\sqrt{n+2}-\sqrt{n+1}}{\sqrt{n+1}-\sqrt{n}}(x-5)\right|$

$\Rightarrow R = 1$

endpoints: $x=4 \to$ conv

$x = 6 \to$ divg

(a) [2,8)
(b) (4,6)
(c) (4,6]
(d) [4,6)
(e) (2,8]
(f) none of the above

10) $\frac{1}{1\cdot 2} - \frac{1}{3\cdot 2^3} + \frac{1}{5\cdot 2^5} - \frac{1}{7\cdot 2^7} + \cdots\cdots =$

$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

let $x = 1/2$

$\tan^{-1} \frac{1}{2} = \frac{1}{2} - \frac{1}{3\cdot 2^3} + \frac{1}{5\cdot 2^5} - \frac{1}{7\cdot 2^7} + \cdots$

(a) $\tan^{-1}(\frac{1}{2})$
(b) $\ln(\frac{5}{2})$
(c) $e^{-1/2}$
(d) $\sin(\frac{3}{2})$
(e) $-\frac{3}{2}\cos(\frac{3}{2})$
(f) none of the above

11) The radius of convergence of the power series $\sum_{n=0}^{\infty}[1\cdot 3\cdot 5\cdot 7\cdots(2n+1)]\ (2x-5)^{2n}$ equals

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\cancel{1\cdot 3\cdot 5\cdot 7\cdots}(2n+3)(2x-5)^{2n+2}}{\cancel{1\cdot 3\cdot 5\cdots(2n+1)}(2x-5)^{2n}}\right|$

$= (2n+3)|2x-5|^2 \to \infty$ as $n \to \infty$

$\Rightarrow R = 0$

(a) ∞
(b) 5/2
(c) 1
(d) 0
(e) π
(f) none of the above

12) The sum of the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n3^{n+1}}$ is equal to

$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$

let $x = 1/3$

$\ln(4/3) = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-1)^{-1}}{n 3^n}$

$-\ln(4/3) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n 3^n}$

multiply by $\frac{1}{3}$

$$\boxed{-\frac{1}{3}\ln(4/3) = \sum_{n=1}^{\infty}\frac{(-1)^n}{n 3^{n+1}} \qquad \ln\sqrt[3]{3/4} =}$$

(a) $\ln(4/3)$
(b) $-\ln(4/3)$
(c) $1/3 - \ln(4/3)$
(d) $1/9 - \ln\sqrt[3]{4/3}$
(e) $1/9 - \ln\sqrt[9]{4/3}$
(f) none of the above

13) $(e-2) - \frac{(e-2)^2}{2} + \frac{(e-2)^3}{3} - \frac{(e-2)^4}{4} + \frac{(e-2)^5}{5} - \cdots\cdots =$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

let $x = e-2$

$\ln(e-1) = (e-2) - \frac{(e-2)^2}{2} + \frac{(e-2)^3}{3} - \frac{(e-2)^4}{4} + \cdots$

(a) $\ln(e-1)$
(b) $\ln(1-e)$
(c) $\ln(e-2)$
(d) $\ln(2-e)$
(e) $2\ln(e-1)$
(f) none of the above

#9) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = |x-5| \cdot \lim_{n\to\infty}\frac{\sqrt{n+2}-\sqrt{n+1}}{\sqrt{n+1}-\sqrt{n}} = |x-5| < 1 \Rightarrow R = 1$

4 5 6

$x = 6 \Rightarrow \sum_{n=1}^{\infty}(\sqrt{n+1}-\sqrt{n}) = -\sum_{n=1}^{\infty}(\sqrt{n}-\sqrt{n+1})$ divg telescoping

$x = 4 \Rightarrow \sum_{n=1}^{\infty}(\sqrt{n+1}-\sqrt{n})(-1)^n \Rightarrow$ alt + lim = 0 + dec $\Rightarrow$ conv

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MATH-102 Term-152 CQ11 Form B

(show all your work and circle one letter to get a full mark or you will get zero)

1) The sequence $\left\{\frac{10}{4}-\sqrt{2},\ \frac{19}{8}-\sqrt{2\sqrt{2}},\ \frac{28}{12}-\sqrt{2\sqrt{2\sqrt{2}}},\ \frac{37}{16}-\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}},\ \ldots\ldots\right\}$

(a) Converges to 7/4

(b) Converges to 5/4

(c) Converges to 3/4

(d) Converges to 1/4

(e) is divergent

(f) None of the above

$$\left\{\frac{10}{4}, \frac{19}{8}, \frac{28}{12}, \frac{37}{16}, \ldots\right\} = \left\{\frac{9n+1}{4n}\right\}_{n=1}^{\infty}$$

$$\lim_{n\to\infty} \frac{9n+1}{4n} = \frac{9}{4}$$

$$\left\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots\right\}$$

$$a_{n+1} = \sqrt{2a_n} \quad n = 1, 2, 3, \ldots$$

$$a_1 = \sqrt{2}$$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \sqrt{2a_n}$$

$$\lim_{n\to\infty} a_{n+1} = \sqrt{2 \lim_{n\to\infty} a_n}$$

$$L = \sqrt{2L}$$

$$\Rightarrow L^2 = 2L$$

$$\Rightarrow L(L-2) = 0$$

$$\Rightarrow L = 2$$

Hence, the limit of the sequence is

$$\frac{9}{4} - 2 = \frac{9}{4} - \frac{8}{4} = \frac{1}{4}$$