Name: KEY	ID: KEY	Sec: 10

MATH-102 Term-132 CQ10 (show all your work & write final answer in the box)

1) $\int \frac{x^2}{(4-x^2)^{3/2}}\,dx$ = I

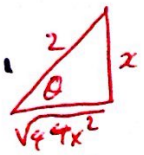
$x = 2\sin\theta \to dx = 2\cos\theta\,d\theta$

$(4-x^2)^{3/2} = (2\cos\theta)^3 = 8\cos^3\theta$

$I = \int \frac{4\sin^2\theta}{8\cos^3\theta}(2\cos\theta\,d\theta)$

$= \int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta = \int \tan^2\theta\,d\theta$

$= \int (\sec^2\theta - 1)\,d\theta$



$= \tan\theta - \theta + C = \frac{x}{\sqrt{4-x^2}} - \sin^{-1}\frac{x}{2} + C$

$\int_0^{\sqrt{3}} \frac{x^2}{(4-x^2)^{3/2}}\,dx =$

$= \left[\frac{x}{\sqrt{4-x^2}} - \sin^{-1}\frac{x}{2}\right]_0^{\sqrt{3}}$

$= \left(\frac{\sqrt{3}}{\sqrt{3}} - \sin^{-1}\frac{1}{2}\right) - \left(\sin^{-1}0\right)$

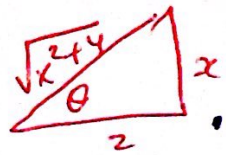
$= \frac{\sqrt{3}}{\sqrt{3}} - \pi/6$

Ans: $\operatorname{sech} x =$

2) $\int \frac{1}{x\sqrt{x^2+4}}\,dx$

$x = 2\tan\theta \to dx = 2\sec^2\theta\,d\theta$

$\sqrt{x^2+4} = 2\sec\theta$



$I = \int \frac{2\sec^2\theta\,d\theta}{(2\tan\theta)(2\sec\theta)} = \frac{1}{2}\int \frac{\sec\theta\,d\theta}{\tan\theta}$

$= \frac{1}{2}\int \frac{1}{\sin\theta}\,d\theta = \frac{1}{2}\int \csc\theta\,d\theta$

$= \frac{1}{2}\ln|\csc\theta - \cot\theta| + C$

$= \frac{1}{2}\ln\left|\frac{\sqrt{x^2+4}}{x} - \frac{2}{x}\right| + C$

Ans: $y' =$