(circle one letter only)

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1) Using four rectangles and the midpoint rule, the area under the graph of $f(x) = 1 + x^2$ from $x = 0$ to $x = 4$ is approximately equal to
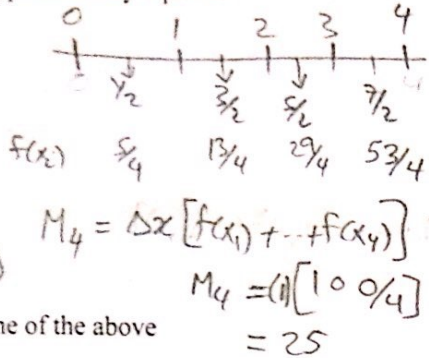
(a) 27

(b) 20

(c) 30

(d) 18

(e) 25 ⟵ circled

(f) None of the above

Handwritten:

$M_4 = \Delta x [f(x_1) + \cdots + f(x_4)]$

$M_4 = (1)[100\%/4]$

$= 25$

Number line from 0 to 4 with midpoints $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$ and $f(x)$ values $\frac{5}{4}, \frac{13}{4}, \frac{29}{4}, \frac{53}{4}$

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2) $\int_0^1 \frac{e^{\tan^{-1}x}}{1+x^2}dx = I$

(a) $1 - e^{\pi/2}$

(b) $e^\pi - \pi$

(c) $e^{\pi/2} - 1$

(d) $e^{\pi/4} - 1$ ⟵ circled

(e) $e^{-\pi/4} - 2$

(f) None of the above

Handwritten:

$u = \tan^{-1}x \Rightarrow du = \frac{dx}{1+x^2}$

$I = \int_0^{\pi/4} e^u du = [e^u]_0^{\pi/4}$

$= e^{\pi/4} - 1$

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3) $\int_0^1 (1+x)\sqrt{4-4x}\,dx = 2\int_0^1 (1+x)\sqrt{1-x}\,dx$

(a) 14/15

(b) 15/28

(c) 28/15 ⟵ circled

(d) 28/13

(e) 15/13

(f) None of the above

Handwritten:

let $u = 1-x \Rightarrow du = -dx$

$x = 1 - u$

$I = -2\int_1^0 (2-u)u^{1/2}\,du$

$= 2\int_0^1 (2u^{1/2} - u^{3/2})\,du$

$= 2\left[\frac{4}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_0^1$

$= 2\left[\frac{4}{3} - \frac{2}{5}\right] = 2\left[\frac{20-6}{15}\right]$

$= 28/15$

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4) If g is a continuous function such that $\int_0^{2x} e^{t/2}g(t)dt = \frac{1}{2}xe^x$, then g(4) =

(a) 4/3

(b) 3/4 ⟵ circled

(c) 3/2

(d) 4/5

(e) 2/3

(f) None of the above

Handwritten:

diff both sides, use FTC

$e^x g(2x)(2) = \frac{1}{2}[e^x + xe^x]$

$g(2x) = \frac{1}{4}[1 + x]$

let $x = 2$

$g(4) = \frac{1}{4}[3] = 3/4$

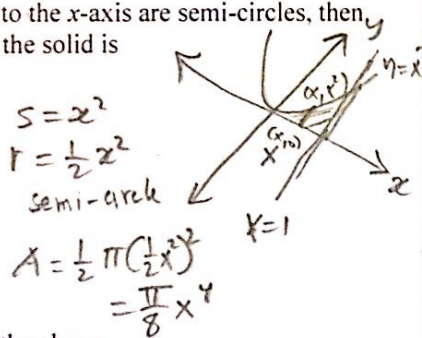
5) The base of a solid is bounded by the curves $y = x^2$, $y = 0$ and $x = 1$. If the cross-sections perpendicular to the x-axis are semi-circles, then the volume of the solid is

(a) $4/\pi$

(b) $40/\pi$

(c) $\pi/4$

(d) $\pi/40$ ⟵ circled

(e) 2π

(f) None of the above

Handwritten:

$S = x^2$

$r = \frac{1}{2}x^2$

semi-circle

$A = \frac{1}{2}\pi\left(\frac{1}{2}x^2\right)^2$

$= \frac{\pi}{8}x^4$

$V = \frac{\pi}{8}\int_0^1 x^4\,dx = \frac{\pi}{8}\left(\frac{1}{5}\right) = \frac{\pi}{40}$

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6) The area of the region enclosed by the curves $y^2 = -x$ and the line $x + y + 2 = 0$ is equal to
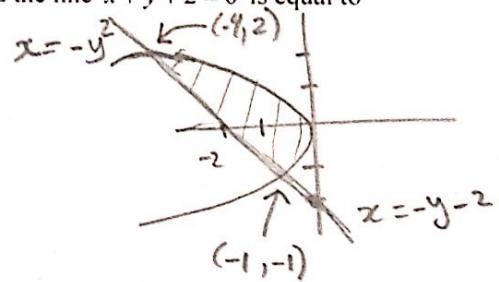
(a) 9/2 ⟵ circled

(b) 7/2

(c) 1

(d) 0

(e) 5/2

(f) None of the above

Handwritten:

$x = -y^2$, $(-4,2)$

$x = -y-2$, $(-1,-1)$

Right to Left

write all curves $x = \cdots$

$A = \int_{-1}^{2}[(-y^2) - (-y-2)]\,dy$

$= \left[-\frac{1}{3}y^3 + \frac{1}{2}y^2 + 2y\right]_{-1}^{2} = \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$

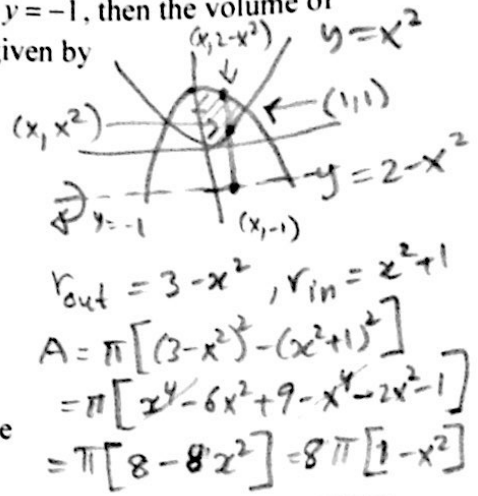$= 8 - \frac{8}{3} - \frac{1}{3} - \frac{1}{2} = 5 - \frac{1}{2} = 9/2$

7) $\int_{-2}^{2}\left(\dfrac{x^2 \tan x}{x^2+1} + \sqrt{4-x^2}\right)dx =$

(a) 0

(b) $\pi/4$

(c) $\pi/3$

(d) π

(e) $\pi/2$

(f) None of the above ⟵ *(circled)*

Handwritten work:

$f(x) = \dfrac{x^2 \tan x}{x^2+1}$ is odd

$f(-x) = \dfrac{x^2 \tan(-x)}{x^2+1} = \dfrac{-x^2 \tan x}{x^2+1}$

$= -f(x) \Rightarrow \int_{-2}^{2} f(x)\,dx = 0$

$\int_{-2}^{2}\sqrt{4-x^2}\,dx$ half circle with $r=2$

$\int_{-2}^{2}\sqrt{4-x^2}\,dx = 2\pi$

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8) The region in the first quadrant enclosed by the parabolas $y = x^2$, $y = 2 - x^2$, and the y-axis is rotating about the line $y = -1$, then the volume of the solid generated is given by

(a) $\int_{0}^{1}\pi(1-x^4)\,dx$

(b) $\int_{0}^{2}8\pi(1+x^2)\,dx$

(c) $\int_{0}^{1}4\pi(1-x^2)\,dx$

(d) $\int_{0}^{1}8\pi(1-x^2)\,dx$ *(circled)*

(e) $\int_{0}^{1}4\pi(1+x^2)\,dx$

(f) None of the above

Handwritten work:

$(x, 2-x^2)$, $y = x^2$ ⟵ $(1,1)$

(x, x^2), $y = 2-x^2$, $y = -1$, $(x, -1)$

$r_{out} = 3 - x^2$, $r_{in} = x^2 + 1$

$A = \pi\left[(3-x^2)^2 - (x^2+1)^2\right]$

$= \pi\left[x^4 - 6x^2 + 9 - x^4 - 2x^2 - 1\right]$

$= \pi\left[8 - 8x^2\right] = 8\pi\left[1 - x^2\right]$

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9) If f is an even function such that $\int_{-3}^{3} f(t)\,dt = 10$ and $\int_{-2}^{2} f(t)\,dt = 16$, then $\int_{\sqrt{2}}^{\sqrt{3}} 2x f(x^2)\,dx = I$

(a) 0

(b) 3

(c) -3 *(circled)*

(d) 8

(e) -5

(f) None of the above

Handwritten work:

f even $\Rightarrow \int_{0}^{2} = 8,\ \int_{0}^{3} = 5$

$u = x^2 \Rightarrow du = 2x\,dx$

$I = \int_{2}^{3} f(u)\,du = 5 - 8 = -3$

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10) If $F(x) = \int_{1/2}^{2x} f(t)\,dt$ and $f(t) = \int_{1/2}^{t^2}\dfrac{\sqrt{1+u^2}}{u}\,dt$ then $F''(1) =$

(a) $5\sqrt{17}$

(b) $4\sqrt{17}$ *(circled)*

(c) $3\sqrt{17}$

(d) $2\sqrt{17}$

(e) $\sqrt{17}$

(f) None of the above

Handwritten work:

$F'(x) = f(2x)(2)$

$F''(x) = f'(2x)(2)(2)$

$= 4 f'(2x)$

$f'(t) = \dfrac{\sqrt{1+t^4}}{t^2}(2t) = \dfrac{2}{t}\sqrt{1+t^4}$

$f'(2x) = \dfrac{2}{2x}\sqrt{1+16x^4} = \dfrac{1}{x}\sqrt{1+16x^4}$

$F''(1) = 4 f'(2) = (4)\sqrt{17}$

$= 4\sqrt{17}$

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11) $\int_{0}^{\frac{\pi}{4}}(\sec x + \cos x)^2\,dx = I$

(a) $(5\pi+10)/8$ *(circled)*

(b) $(4\pi+10)/8$

(c) $(3\pi+10)/8$

(d) $(5\pi+10)/8$ *(circled)*

(e) $(2\pi+10)/8$

(f) None of the above

Handwritten work:

$(\sec x + \cos x)^2 =$

$\sec^2 x + 2 + \cos^2 x$

$\sec^2 x + 2 + \frac{1}{2} + \frac{1}{2}\cos 2x$

$I = \left[\tan x + \frac{5}{2}x + \frac{1}{4}\sin 2x\right]_{0}^{\frac{\pi}{4}}$

$= \left(1 + \frac{5\pi}{8} + \frac{1}{4}\right) - (0 + 0 + 0)$

$= \frac{5}{4} + \frac{5\pi}{8} = \frac{10}{8} + \frac{5\pi}{8}$

$= (10 + 5\pi)/8$

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12) $\int \dfrac{\ln(\tan^{-1}x)}{(x^2+1)\tan^{-1}x}\,dx = I$

(a) $\frac{1}{2}\left[\ln(\tan^{-1}x)\right] + C$

(b) $\frac{1}{2}\left[\ln(\tan^{-1}x)\right]^2 + C$ *(circled)*

(c) $\left[\ln(\tan^{-1}x)\right]^2 + C$

(d) $\frac{1}{2}\left[\tan^{-1}x\right]^2 + C$

(e) $\ln(\tan^{-1}x) + C$

(f) None of the above

Handwritten work:

$u = \tan^{-1}x \Rightarrow du = \dfrac{dx}{1+x^2}$

$I = \int \dfrac{\ln u}{u}\,du$

$\omega = \ln u \Rightarrow d\omega = \dfrac{du}{u}$

$I = \int \omega\,d\omega = \frac{1}{2}\omega^2 + C$

$= \frac{1}{2}(\ln u)^2 + C$

$= \frac{1}{2}\left[\ln(\tan^{-1}x)\right]^2 + C$