Name: KEY	ID: KEY	Sec8(9:00-9:50) Sec9(10:00-10:50)

MATH-102 **Term-152** **Q1**

Show all your work and circle one answer

1) $\sum_{i=1}^{n} \frac{6(1+i)^2}{n} =$

(a) $2n^2+9n+12$
(b) $n^2+9n+12$
(c) $n^2+8n+12$
(d) $2n^2+13$
(e) $2n^2+9n+13$
(f) none of the above

$$\frac{6}{n}\sum_{i=1}^{n}(1+i)^2 = \frac{6}{n}\sum_{i=1}^{n}(i^2+2i+1)$$

$$= \frac{6}{n}\sum_{i=1}^{n} i^2 + \frac{6}{n}\sum_{i=1}^{n} 2i + \frac{6}{n}\sum_{i=1}^{n} 1$$

$$= \frac{\cancel{6}}{\cancel{n}}\left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}\right) + \frac{6}{\cancel{n}}(\cancel{2})\frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{6}{n}(n)$$

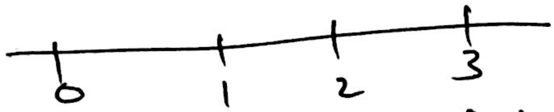
$$= (n+1)(2n+1) + 6(n+1) + 6$$

$$= 2n^2+3n+1+6n+6+6 = 2n^2+9n+13$$

2) The area under the graph of $f(x)=\frac{6}{x+1}$ from $x=0$ to $x=3$ to using three rectangles and right endpoints is approximately equal to

(a) 8
(b) 9
(c) 10
(d) 11
(e) 12
(f) none of the above

$a=0$, $b=3$, $n=3$, $\Delta x = \frac{3}{3} = 1$



$f(1) = 3$ $f(2) = 2$ $f(3) = 3/2$

$$R_3 = (3)(1) + (2)(1) + \left(\tfrac{3}{2}\right)(1)$$
$$= 5 + \tfrac{3}{2} = 13/2$$

3) $\lim_{n\to\infty}\left(\sum_{i=1}^{n}\sqrt{\frac{1}{n^2}-\frac{i^2}{n^4}}\right) =$

(a) $\frac{1}{4}$
(b) $\frac{\pi}{4}$
(c) 1
(d) $\frac{e}{2}$
(e) π
(f) none of the above

$$R_n = \sum_{i=1}^{n}\sqrt{\frac{1}{n^2}-\frac{i^2}{n^4}}$$

$a=0$, $b=1$, $\Delta x = \frac{1}{n}$
$x_i = i/n$

$$= \sum_{i=1}^{n}\sqrt{\frac{1}{n^2}\left(1-\frac{i^2}{n^2}\right)}$$

$$= \sum_{i=1}^{n}\frac{1}{n}\sqrt{1-i^2/n^2} = \sum_{i=1}^{n}\Delta x f(x_i)$$

$$f(x) = \sqrt{1-x^2}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{\frac{1}{n^2}-\frac{i^2}{n^4}} = \int_0^1\sqrt{1-x^2}\,dx = \frac{\pi}{4}$$