Math 102

Quiz#3

Name:

Serial No.:

1. Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

. The requirement is that $\frac{1}{b-0}\int_0^b f(x)\,dx = 3$. The LHS of this equation is equal to

$\frac{1}{b}\int_0^b (2+6x-3x^2)\,dx = \frac{1}{b}\left[2x+3x^2-x^3\right]_0^b = 2+3b-b^2$, so we solve the equation $2+3b-b^2=3 \Leftrightarrow$

$b^2-3b+1=0 \quad \Leftrightarrow \quad b = \frac{3 \pm \sqrt{(-3)^2-4\cdot 1\cdot 1}}{2\cdot 1} = \frac{3\pm\sqrt{5}}{2}$. Both roots are valid since they are positive.

2. Evaluate $I = \int_4^9 \frac{\ln x}{\sqrt{x}}\,dx$.

. Let $u = \ln y$, $dv = \frac{1}{\sqrt{y}}\,dy = y^{-1/2}\,dy \quad \Rightarrow \quad du = \frac{1}{y}\,dy$, $v = 2y^{1/2}$. Then

$$\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy = \left[2\sqrt{y}\,\ln y\right]_4^9 - \int_4^9 2y^{-1/2}\,dy = (6\ln 9 - 4\ln 4) - \left[4\sqrt{y}\right]_4^9 = 6\ln 9 - 4\ln 4 - (12-8)$$

$$= 6\ln 9 - 4\ln 4 - 4$$

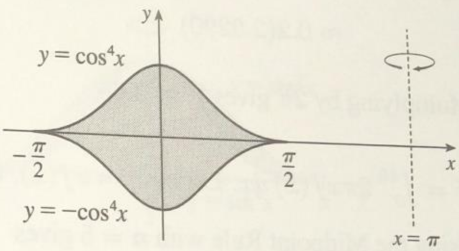
3. The region bounded by $y = \cos^4 x$, $y = -\cos^4 x$ and in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ is rotating about $x = \pi$. Find the volume of the resulted solid.

(Just set up the integration formula)

$$V = 2\pi \int_{-\pi/2}^{\pi/2} (\pi - x)[\cos^4 x - (-\cos^4 x)]\,dx$$

$$= 4\pi \int_{-\pi/2}^{\pi/2} (\pi - x)\cos^4 x\,dx$$

$V \approx 46.50942$



4. The region bounded by $y = x^3$, $y = 0$ and $x = 1$ is rotating about $y = 1$. Find the *volume* of the resulted solid.

The shell has radius $1 - y$, circumference $2\pi(1 - y)$, and height $1 - \sqrt[3]{y}$ $\left[y = x^3 \quad \Leftrightarrow \quad x = \sqrt[3]{y}\right]$.

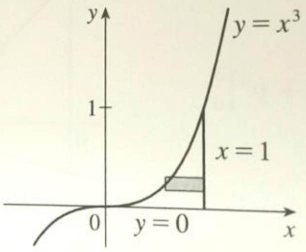
$V = \int_0^1 2\pi(1 - y)(1 - y^{1/3})\, dy$

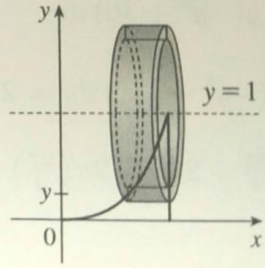
$= 2\pi \int_0^1 (1 - y - y^{1/3} + y^{4/3})\, dy$

$= 2\pi \left[y - \frac{1}{2}y^2 - \frac{3}{4}y^{4/3} + \frac{3}{7}y^{7/3}\right]_0^1$

$= 2\pi\left[\left(1 - \frac{1}{2} - \frac{3}{4} + \frac{3}{7}\right) - 0\right]$

$= 2\pi\left(\frac{5}{28}\right) = \frac{5}{14}\pi$





5. Evaluate $I = \int \ e^{-\theta} \cos 2\theta \, d\theta$.

First let $u = e^{-\theta}$, $dv = \cos 2\theta \, d\theta \quad \Rightarrow \quad du = -e^{-\theta}\, d\theta$, $v = \frac{1}{2}\sin 2\theta$. Then

$I = \int e^{-\theta} \cos 2\theta \, d\theta = \frac{1}{2}e^{-\theta}\sin 2\theta - \int \frac{1}{2}\sin 2\theta \left(-e^{-\theta}\, d\theta\right) = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\int e^{-\theta}\sin 2\theta \, d\theta.$

Next let $U = e^{-\theta}$, $dV = \sin 2\theta \, d\theta \quad \Rightarrow \quad dU = -e^{-\theta}\, d\theta$, $V = -\frac{1}{2}\cos 2\theta$, so

$\int e^{-\theta}\sin 2\theta \, d\theta = -\frac{1}{2}e^{-\theta}\cos 2\theta - \int\left(-\frac{1}{2}\right)\cos 2\theta\left(-e^{-\theta}\, d\theta\right) = -\frac{1}{2}e^{-\theta}\cos 2\theta - \frac{1}{2}\int e^{-\theta}\cos 2\theta \, d\theta.$

So $I = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\left[\left(-\frac{1}{2}e^{-\theta}\cos 2\theta\right) - \frac{1}{2}I\right] = \frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta - \frac{1}{4}I \quad \Rightarrow$

$\frac{5}{4}I = \frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta + C_1 \quad \Rightarrow \quad I = \frac{4}{5}\left(\frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta + C_1\right) = \frac{2}{5}e^{-\theta}\sin 2\theta - \frac{1}{5}e^{-\theta}\cos 2\theta + C.$