King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Term 152
Tuesday 17/05/2016
Net Time Allowed: 180 minutes

MASTER VERSION

1. The definite integral $\int_{-1}^{5} 3x + 2\,dx$ is equal to

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n}\left(\frac{18i}{n} - 1\right)$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n}\left(\frac{6i}{n} + 1\right)$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{18i}{n} - 4\right)$

(d) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{6i}{n} - 3\right)$

(e) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n}\left(\frac{18i}{n} + 5\right)$

$f(x) = 3x+2$, $a=-1$, $b=5$

$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$, $x_i = a + i\Delta x$, $\Delta x = \frac{b-a}{n}$

$$\int_{-1}^{5} (3x+2)dx = \lim_{n\to\infty} \sum_{i=1}^{n} (3x_i+2)\left(\frac{5+1}{n}\right)$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n}\left(3\left(-1+\frac{6}{n}i\right)+2\right)$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n}\left(\frac{18}{n}i - 1\right).$$

2. Let A be the exact area below a curve over the interval $[0,1]$ and let B be an estimation of the same area using left endpoints of 10 subintervals . For which one of the following curves B is greater than A?

(a) $\cos x$

(b) $\sin x$

(c) e^x

(d) x

(e) $\sqrt{x}$

For $B = L_{10}$ to be greater than A, the function must be decreasing on $[0,1]$.

Hence, the curve is $\cos x$.

The other curves are for increasing functions.

3. $\int \sin^3 x \cos^2 x\, dx = \int \sin^2 x \cos^2 x \sin x\, dx = \int (1-\cos^2 x)\cos^2 x \sin x\, dx$

$= \int (\cos^2 x - \cos^4 x) \sin x\, dx$

$= \int (u^2 - u^4)(-du)$, $u = \cos x$, $du = -\sin x\, dx$

$= -\frac{u^3}{3} + \frac{u^5}{5} + C$

$= \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$.

(a) $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$

(b) $\frac{1}{4}\cos^4 x - \frac{1}{3}\sin^3 x + C$

(c) $\frac{1}{4}\cos^4 x - \frac{1}{6}\cos^6 x + C$

(d) $\frac{1}{3}\cos^3 x + \frac{1}{2}\sin^2 x + C$

(e) $-\cos^3 x \sin^2 x + C$

4. $\int_0^{\pi/4} \tan x \ln(\cos x) dx = I$

Let $u = \ln(\cos x) \Rightarrow du = -\frac{\sin x}{\cos x}dx = -\tan x\, dx$

$x = 0 \Rightarrow u = \ln(1) = 0$

$x = \frac{\pi}{4} \Rightarrow u = \ln(1/\sqrt{2}) = -\frac{1}{2}\ln 2$

$I = \int_0^{-\frac{1}{2}\ln 2} u(-du) = -\frac{u^2}{2}\Big]_0^{-\frac{1}{2}\ln 2}$

$= -\frac{\frac{1}{4}(\ln 2)^2}{2} + \frac{0}{2}$

$= -\frac{1}{8}(\ln 2)^2$.

(a) $-\frac{1}{8}(\ln 2)^2$

(b) $\sqrt{2}\ln 2$

(c) $2\sqrt{2}$

(d) -1

(e) 0

5. If g is a continuous function so that

$\int_{\pi}^{2x} \cos\left(\frac{t}{2}\right) g(t)dt = \frac{x}{2}\sin x - \frac{\pi}{4}$, then $g(2\pi) =$

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) $-\pi$

(d) $\frac{1+\pi}{2}$

(e) $\frac{-1+\pi}{4}$

$$\frac{d}{dx}\left(\int_{\pi}^{2x} \cos\left(\frac{t}{2}\right) g(t)\,dt\right) = \frac{d}{dx}\left(\frac{x}{2}\sin x - \frac{\pi}{4}\right)$$

$$\Rightarrow \cos x \, g(2x) \cdot 2 = \frac{x}{2}\cos x + \frac{1}{2}\sin x$$

$$\Rightarrow g(2x) = \frac{x}{4} + \frac{1}{4}\tan x$$

Thus, $g(2\pi) = \frac{\pi}{4} + \frac{1}{4}\tan\pi$

$= \frac{\pi}{4} + \frac{1}{4}(0)$

$= \pi/4$.

6. The area enclosed by the curves $x = 2y^2$ and $x = 4 + y^2$ is equal to

(a) $\frac{32}{3}$

(b) $\frac{31}{3}$

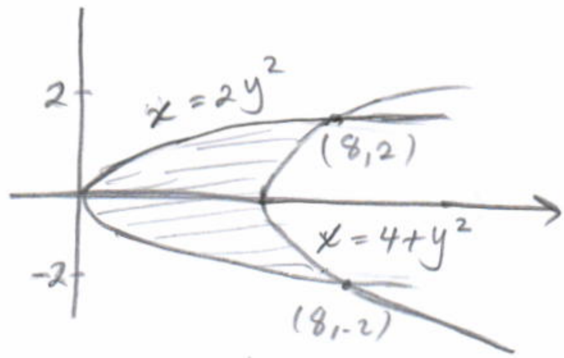
(c) $\frac{29}{3}$

(d) $\frac{28}{3}$

(e) $\frac{26}{3}$

$$2y^2 = 4 + y^2 \Rightarrow y^2 = 4 \Rightarrow y = -2,\ y = 2$$

$$A = \int_{-2}^{2} (4 + y^2 - 2y^2)\,dy$$



$$= \int_{-2}^{2} (4 - y^2)\,dy$$

$$= 2\int_{0}^{2} (4 - y^2)\,dy = 2\left(4y - \frac{1}{3}y^3\right)\Big]_0^2$$

$$= 16 - \frac{16}{3} - 0 = \frac{32}{3}.$$

7. The improper integral $\int_1^\infty \frac{e^{1/x}}{x^2}dx$ $= \lim_{t\to\infty} \int_1^t \frac{e^{1/x}}{x^2}dx$

(a) converges to $e-1$.

(b) converges to $1-\frac{1}{e}$.

(c) converges to e.

(d) converges to 1.

(e) diverges.

$$= \lim_{t\to\infty} \int_1^{1/t} e^u(-du), \quad u = \frac{1}{x}, \; du = -\frac{1}{x^2}dx$$

$$= \lim_{t\to\infty} \left[-e^u\right]_1^{1/t}$$

$$= \lim_{t\to\infty} \left(-e^{\frac{1}{t}} + e^1\right)$$

$$= (-e^0 + e) = e - 1.$$

8. The area of the surface obtained by revolving the curve $y = \ln(\sec x)$, $0 \le x \le \pi/3$ about the y-axis is

(a) $2\pi \int_0^{\pi/3} x \sec x \, dx$

(b) $2\pi \int_0^{\pi/3} \ln(\sec x) \sec x \, dx$

(c) $2\pi \int_0^{\pi/3} \sec x \, dx$

(d) $2\pi \int_0^{\pi/3} x \tan x \, dx$

(e) $2\pi \int_0^{\pi/3} \ln(\sec x) \tan x \, dx$

$$S = \int 2\pi x \, ds,$$

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}dx,$$

$$\frac{dy}{dx} = \frac{\sec x \tan x}{\sec x} = \tan x$$

Therefore,

$$S = \int_0^{\pi/3} 2\pi x \sqrt{1+\tan^2 x}\, dx$$

$$= \int_0^{\pi/3} 2\pi x \sqrt{\sec^2 x}\, dx$$

$$= 2\pi \int_0^{2\pi} x \sec x \, dx, \quad 0 \le x \le \frac{\pi}{3}.$$

9. If the velocity of a moving particle is $v(t) = t^2 + 5t - 6$ in m/s, then the total distance travelled by the particle during the time interval $0 \leq t \leq 4$ is

(a) $\int_1^4 t^2 + 5t - 6\,dt - \int_0^1 t^2 + 5t - 6\,dt$

(b) $\int_0^1 t^2 + 5t - 6\,dt - \int_1^4 t^2 + 5t - 6\,dt$

(c) $\int_1^4 t^2 + 5t - 6\,dt$

(d) $\int_0^4 t^2 + 5t - 6\,dt$

(e) $\int_0^1 t^2 + 5t - 6\,dt$

$v(t) = t^2 + 5t - 6$
$= (t+6)(t-1)$, $t \in [0, 4]$.

So, $v(t) \geq 0$ on $[1, 4]$, and $v(t) \leq 0$ on $[0, 1]$

Hence,

Total distance travelled

$$= \int_0^4 |v(t)|\,dt$$

$$= -\int_0^1 (t^2 + 5t - 6)\,dt + \int_1^4 (t^2 + 5t - 6)\,dt.$$

10. The volume generated by rotating the region bounded by $y = \ln x$, $x = e$, and $y = 0$ about the y-axis is

Using Washers:

(a) $\frac{\pi}{2}(e^2 + 1)$

(b) $\frac{\pi}{18}(2e^3 + 1)$

(c) $\frac{\pi}{2}(2e^2 - 1)$

(d) $\frac{\pi}{18}(e^3 - 1)$

(e) $\pi(2e^2 + 1)$

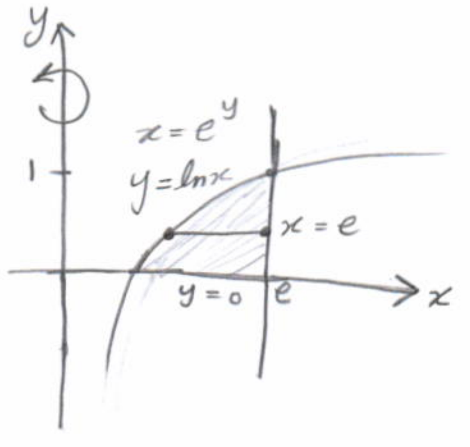
$r_{out} = e$, $r_{in} = e^y$

$A(y) = \pi (e)^2 - (e^y)^2$
$= \pi (e^2 - e^{2y})$

$$V = \int_0^1 A(y)\,dy$$

$$= \pi \int_0^1 (e^2 - e^{2y})\,dy = \pi \left[e^2 y - \tfrac{1}{2} e^{2y} \right]_0^1$$

$$= \pi \left(e^2 - \tfrac{1}{2} e^2 - 0 + \tfrac{1}{2} e^0 \right) = \frac{\pi}{2}(e^2 + 1).$$



11. $\displaystyle\int_0^{\pi/2} \sinh x \sin x\,dx =$ I. let $u = \sinh x$ $\quad dv = \sin x\,dx$

$du = \cosh x\,dx \quad v = -\cos x$

(a) $\frac{1}{2}\cosh\left(\frac{\pi}{2}\right)$

(b) $2\sinh\left(\frac{\pi}{2}\right)$

(c) $\frac{1}{2}\left[\cosh\left(\frac{\pi}{2}\right) - \sinh\left(\frac{\pi}{2}\right)\right]$

(d) $2\left[\cosh\left(\frac{\pi}{2}\right) + \sinh\left(\frac{\pi}{2}\right)\right]$

(e) 0

$$I = -\sinh x \cos x\Big]_0^{\pi/2} + \int_0^{\pi/2} \cosh x \cos x\,dx$$

Let $u = \cosh x \quad dv = \cos x\,dx$

$du = \sinh x \quad V = \sin x$

$$I = -0 + \underbrace{\sinh 0}_{=0} + \cosh x \sin x\Big]_0^{\pi/2} - I$$

$$\Rightarrow 2I = \cosh\left(\tfrac{\pi}{2}\right) - 0$$

$$\Rightarrow I = \tfrac{1}{2}\cosh\left(\tfrac{\pi}{2}\right).$$

12. $\displaystyle\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x^2}\,dx =$ I.

(a) $\ln(\sqrt{2}+1) - \frac{\sqrt{2}}{2}$

(b) $\ln(\sqrt{3}+2) - \frac{\sqrt{3}}{2}$

(c) $\ln(\sqrt{2}) + \frac{\sqrt{2}}{2}$

(d) $\ln(\sqrt{3}) + \frac{\sqrt{3}}{2}$

(e) $\ln(\sqrt{2}+2) - \frac{\sqrt{2}}{2}$

Let $x = \sec\theta$, $0 \le \theta < \frac{\pi}{2}$

$dx = \sec\theta \tan\theta\,d\theta$

$\sqrt{x^2-1} = \tan\theta$

x	1	$\sqrt{2}$
θ	0	$\pi/4$

$$I = \int_0^{\pi/4} \frac{\tan\theta}{\sec^2\theta} \sec\theta \tan\theta\,d\theta$$

$$= \int_0^{\pi/4} \frac{\tan^2\theta}{\sec\theta}\,d\theta = \int_0^{\pi/4} \frac{\sec^2\theta - 1}{\sec\theta}\,d\theta$$

$$= \int_0^{\pi/4} (\sec\theta - \cos\theta)\,d\theta$$

$$= \Big[\ln|\sec\theta + \tan\theta| - \sin\theta\Big]_0^{\pi/4}$$

$$= \left(\ln|\sqrt{2}+1| - \frac{\sqrt{2}}{2}\right) - \left(\ln|1+0| - 0\right)$$

$$= \ln(\sqrt{2}+1) - \frac{\sqrt{2}}{2}.$$

13. $\int \frac{x+2}{x^2+4} dx =$

$$\int \left(\frac{x}{x^2+4} + \frac{2}{x^2+4} \right) dx$$

(a) $\ln \sqrt{x^2+4} + \tan^{-1}\left(\frac{x}{2}\right) + C$

(b) $\ln |x-2| + C$

(c) $\ln(x^2+4) + 2\tan^{-1} x + C$

(d) $\ln \sqrt{x^2+4} + C$

(e) $\ln(x^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

$$= \frac{1}{2} \int \frac{2x}{x^2+4} dx + 2 \int \frac{1}{x^2+4} dx$$

$$= \frac{1}{2} \ln(x^2+4) + 2 \cdot \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= \ln \sqrt{x^2+4} + \tan^{-1}\left(\frac{x}{2}\right) + C.$$

14. The length of the curve $y = \frac{1}{3} + \frac{4}{3}x^{3/2}$, $0 \le x \le 2$ is

(a) $\frac{13}{3}$

(b) $\frac{13}{2}$

(c) $\frac{52}{3}$

(d) $\frac{26}{3}$

(e) $\frac{15}{2}$

$$\frac{dy}{dx} = \frac{4}{3} \cdot \frac{3}{2} x^{1/2} = 2\sqrt{x}$$

$$L = \int_0^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^2 \sqrt{1+4x}\, dx$$

$$= \frac{1}{4} \frac{2}{3} (1+4x)^{3/2} \Big]_0^2$$

$$= \frac{1}{6}(9)^{3/2} - \frac{1}{6}(1)^{3/2}$$

$$= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}.$$

15. $\lim_{n\to\infty} \frac{1^3+2^3+\ldots+n^3}{n^4-5n} = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} i^3}{n^4-5n}$

(a) $\frac{1}{4}$

(b) $\frac{1}{5}$

(c) 1

(d) 0

(e) does not exist

$$= \lim_{n\to\infty} \frac{\frac{n^2(n+1)^2}{4}}{n^4-5n}$$

$$= \lim_{n\to\infty} \frac{n^2(n^2+2n+1)}{4(n^4-5n)}$$

$$= \lim_{n\to\infty} \frac{n^4+2n^3+n^2}{4n^4-20n} = \frac{1}{4}.$$

16. The series $\sum_{n=1}^{\infty} \frac{6}{9n^2-3n-2}$ is

(a) convergent and its sum is 2.

(b) convergent and its sum is 1.

(c) convergent and its sum is 2/3.

(d) convergent and its sum is 6.

(e) divergent.

Since $\frac{6}{9n^2-3n-2} = \frac{6}{(3n-2)(3n+1)}$

$$= \frac{2}{3n-2} + \frac{-2}{3n+1},$$

then,

$$S_n = \sum_{k=1}^{n} 2\left(\frac{1}{3k-2} - \frac{1}{3k+1}\right)$$

$$= 2\left[\left(1-\cancel{\tfrac{1}{4}}\right)+\left(\cancel{\tfrac{1}{4}}-\cancel{\tfrac{1}{7}}\right) + \left(\cancel{\tfrac{1}{7}}-\cancel{\tfrac{1}{10}}\right)+\cdots + \left(\cancel{\tfrac{1}{3n-5}}-\cancel{\tfrac{1}{3n-2}}\right)+\left(\cancel{\tfrac{1}{3n-2}}-\tfrac{1}{3n+1}\right)\right]$$

$$= 2\left(1-\frac{1}{3n+1}\right)$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} 2\left(1-\frac{1}{3n+1}\right) = 2(1-0) = 2.$$

17. If $s_n = n\sin(1/n)$ is the sequence of partial sums of the series $\sum_{n=1}^{\infty} a_n$, then

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} n \sin\left(\frac{1}{n}\right) = \lim_{\frac{1}{n}\to 0} \frac{\sin\left(\frac{1}{n}\right)}{1/n}.$$

(a) $\lim_{n\to\infty} a_n = 0$.

(b) the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(c) the series $\sum_{n=1}^{\infty} a_n$ is convergent and its sum is 0.

(d) $\lim_{n\to\infty} s_n = 0$.

(e) $\lim_{n\to\infty} a_n$ does not exist.

The series is convergent and its sum is 1.

As $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

18. The series $\sum_{n=1}^{\infty} \frac{n^3}{(n^2+n)^q}$ is convergent for

(a) $q > 2$

(b) $q \geq 2$

(c) $q < 2$

(d) $q \leq 2$

(e) $q = 2$

If $q \leq 0$, then the series is divergent.

For $q > 0$, we have

$$\frac{n^3}{(n^2+n)^q} < \frac{n^3}{(n^2)^q} = \frac{1}{n^{2q-3}}.$$

$\sum_{n=1}^{\infty} \frac{1}{n^{2q-3}}$ is a p-series and it is convergent for $2q-3 > 1$, that is,

$2q > 4 \Rightarrow q > 2$.

19. Which one of the following statements is TRUE for the series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{1+n^2}$, where $f(x) = \frac{\cos^2 x}{1+x^2}$?

Since $f'(x) = \frac{-2(1+x^2)\cos x \sin x - 2x\cos^2 x}{(1+x^2)^2}$

$= \frac{-2\cos x\left((1+x^2)\sin x + x\cos x\right)}{(1+x^2)^2}$

is not negative for all $x \geqslant 1$.

So f is not decreasing on $[1, \infty)$.

(a) The integral test is not applicable because f is not decreasing on $[1, \infty)$.

(b) The series converges by the integral test.

(c) The series diverges by the integral test.

(d) The integral test is not applicable because f is not positive on $[1, \infty)$.

(e) The integral test is not applicable because f is discontinuous on $[1, \infty)$.

20. Let $\sum_{n=1}^{\infty}(-1)^n b_n$, where $b_n > 0$, be an alternating series. If the sequence $\{b_n\}$ converges to a non zero number, then

(a) the series diverges by the test for divergence.

(b) the series diverges by the alternating series test.

(c) the series conditionally converges.

(d) the series absolutely converges.

(e) the series diverges by the root test.

$\lim_{n\to\infty} b_n$ exists and is different from zero.

So $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges by the test for divergence.

21. The series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n+1)!}$

(a) converges by the ratio test.

(b) diverges by the alternating series test.

(c) conditionally converges.

(d) converges by the integral test.

(e) diverges by the test for divergence.

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{((n+1)!)^2}{(2n+3)!} \frac{(2n+1)!}{(n!)^2} \right|$$

$$= \lim_{n\to\infty} \frac{(n+1)^2 \cancel{(n!)^2} \cancel{(2n+1)!}}{(2n+3)(2n+2)\cancel{(2n+1)!}\cancel{(n!)^2}}$$

$$= \lim_{n\to\infty} \frac{n^2+2n+1}{4n^2+10n+6}$$

$$= \frac{1}{4} < 1$$

The series converges by the ratio test.

22. The series $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n^{2/3}}$ $= 1 + 0 - \frac{1}{3^{2/3}} + 0 + \frac{1}{5^{2/3}} + 0 - \frac{1}{7^{2/3}}$

is not an alternating series.

So choice (d) is correct.

(a) converges conditionally.

(b) diverges.

(c) converges absolutely.

(d) is not an alternating series.

(e) diverges by the ratio test.

both choices are accepted as correct answers.

Also,

$$= 1 - \frac{1}{3^{2/3}} + \frac{1}{5^{2/3}} - \frac{1}{7^{2/3}} + \cdots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{(2n-1)^{2/3}}$$

converges by the alternating series test.

But, $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2/3}}$ diverges by the limit comparison test with the divergent series $\sum_{n=1}^{\infty} 1/n^{2/3}$.

23. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{3^n}{n}(2x-1)^n$ is

(a) $\left[\frac{1}{3}, \frac{2}{3}\right)$

(b) $\left(\frac{1}{3}, \frac{2}{3}\right)$

(c) $\left(-\frac{1}{3}, \frac{1}{3}\right)$

(d) $\left(-\frac{1}{3}, \frac{1}{3}\right]$

(e) $(-\infty, \infty)$

$$\lim_{n\to\infty}\left|\frac{3^{n+1}(2x-1)^{n+1}}{n+1}\frac{n}{3^n(2x-1)^n}\right| = \lim_{n\to\infty}\frac{3n|2x-1|}{n+1}$$

$$= 3|2x-1| < 1 \iff |x-\tfrac{1}{2}| < \tfrac{1}{6} \iff \tfrac{1}{3} < x < \tfrac{2}{3}.$$

$\underline{x = \frac{1}{3}}$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ convergent alternating harmonic series.

$\underline{x = \frac{2}{3}}$: $\sum_{n=1}^{\infty} \frac{1}{n}$ divergent harmonic series.

Therefore, the interval of convergence is $\left[\frac{1}{3}, \frac{2}{3}\right)$.

24. For $|x| < 1$, a power series representation of $f(x) = x\tan^{-1}x$ is

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+2}}{2n+1}$

(b) $\sum_{n=0}^{\infty} \frac{x^{2n+2}}{2n+1}$

(c) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$

(d) $\sum_{n=1}^{\infty}(-1)^n \frac{x^{2n+2}}{2n}$

(e) $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{x^{2n+1}}{2n}$

$$\tan^{-1}x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}, \quad |x|<1$$

$$x\tan^{-1}x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+2}}{2n+1}, \quad |x|<1.$$

25. The radius of convergence of the series $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{-n^2} x^n$ is

(a) e

(b) $\frac{1}{e}$

(c) ∞

(d) e^2

(e) $\frac{1}{e^2}$

By the Root Test,

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \left|\left(1+\frac{1}{n}\right)^{-n^2} x^n\right|^{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{-n} |x| = e^{-1}|x|$$

the series converges when $e^{-1}|x| < 1$

$\Longrightarrow |x| < e.$

So, the radius of convergence is e.

26. If $f(x) = \frac{1}{1+x}$ has a power series expansion at $x=2$, then its Taylor series centered at $x=2$ is

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}}(x-2)^n$

(b) $\sum_{n=0}^{\infty} (-1)^n (x-2)^n$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}(x-2)^n$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^n$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n}(x-2)^n$

$f'(x) = \frac{-1}{(1+x)^2}$, $f''(x) = \frac{1.2}{(1+x)^3}$

$f^{(3)}(x) = \frac{-1.2.3}{(1+x)^4}$, $f^{(4)}(x) = \frac{1.2.3.4}{(1+x)^5}$

$, \ldots, f^{(n)}(x) = \frac{(-1)^n n!}{(1+x)^{n+1}}$

So $f^{(n)}(2) = \frac{(-1)^n n!}{3^{n+1}}$ and

the Taylor series centered at $x=2$ is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}}(x-2)^n.$$

27. If $P(x)$ is the sum of the first three non zero terms of the Maclaurin series of $f(x) = (1+x)^{-1/2}\cos x$, then $P(1/2) =$ (Hint: You may use the product of the Maclaurin series of $\cos x$ and $(1+x)^{-1/2}$.)

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{(-\frac{1}{2})(\frac{-3}{2})}{2!}x^2 + \cdots$$

$$\Rightarrow (1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 + \cdots$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots$$

$$(1+x)^{-1/2}\cos x = 1 - \frac{1}{2}x + \frac{3}{8}x^2 + \cdots$$

$$- \frac{x^2}{2} + \cdots$$

$$P(x) = 1 - \frac{1}{2}x + \left(\frac{3}{8} - \frac{1}{2}\right)x^2 \quad \text{(The first three nonzero terms)}$$

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2.$$

So, $P(\frac{1}{2}) = 1 - \frac{1}{4} - \frac{1}{32} = \frac{32-8-1}{32} = \frac{23}{32}$.

(a) $\frac{23}{32}$

(b) $\frac{16}{17}$

(c) $\frac{25}{32}$

(d) $\frac{33}{34}$

(e) $\frac{9}{4}$

28. The sum of the series $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n+1}}{4^n(2n+1)!}$ is

$$= \sum_{n=0}^{\infty}(-1)^n \frac{\pi^{2n+1}}{2^{2n}(2n+1)!}$$

$$= 2\sum_{n=0}^{\infty}(-1)^n \frac{(\pi/2)^{2n+1}}{(2n+1)!}$$

$$= 2\sin(\pi/2) = 2.$$

(a) 2

(b) -2

(c) 1

(d) -1

(e) 0