King Fahd University Of Petroleum & Minerals
Department Of Mathematics And Statistics
STAT501 : Probability and Mathematical Statistics (151)
Second Exam **Thursday November 26, 2015**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	17	
Two	8	
Three	13	
Four	13	
Five	13	
Six	17	
Seven	14	
Eight	15	
Total	110	

Question.1 (**7+3+7=17-Points**)

Suppose that X and Y are two discrete random variables with joint mass function given by the following table:

y	1	3	5	6
x				
x	1/9	1/27	1/27	1/9

(a) Show that $g(x)$ is a PDF.

(b) Find the DF $G(x)$ of the PDF $g(x)$

(c) Show that the $G(x)$ obtained in part (b) is indeed a DF.

Question.2 (**4+4=8-Points**)

(a) Let X be an RV of discrete type with PMF given by:
$f(x) = \binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \ldots, n$, where $0 < p < 1$, n is a positive integer, let $Z = \frac{1}{X+1}$, find the PMF of Z

(b) If X is a discrete RV with PMF given by:

x	-1	0	1	2	3
$P\{X = x\}$	0.2	0.1	0.15	0.25	0.30

Find the PMF of $Y = X^2 - X$

Question.3 (**6+7=13-Points**) Let X be a continuous RV with PDF given by: $f(x) = \begin{cases} 1, 0 < x < 1 \\ 0, \text{ elsewhere} \end{cases}$, find the PDF of the following RVs:

(a) $Y = -\ln(X)$

(b) $U = X^2$

Question.4 (**8+5=13-Points**) Let X be a continuous RV with PDF $f(x) = \begin{cases} \frac{\beta\alpha^{\beta}}{x^{\beta+1}}, x \geq \alpha \\ 0, x < \alpha \end{cases}$

(a) Show that the moments of order n exists if and only if $n < \beta$

(b) Find the mean and variance of X

Question.5 (**6+7=13-Points**) Describe the RV in each of the following cases, that is, write the PMF, then ***find the mean*** of each one:

(a) The PGF of X is given by : $P(s) = \frac{1}{2}(s + s^2)$

(b) The MGF of X is: $M_x(t) = \frac{1}{3} + \frac{1}{4}e^t + \frac{1}{6}e^{2t} + \frac{1}{12}e^{3t} + \frac{1}{6}e^{4t}$

Question .6 **(8+4+5=17-Points)** Let X and Y be discrete RVs with joint PMF given by:

$$P(X=x,Y=y)=\begin{cases}\frac{1}{21}(x+y), x=1,2, y=1,2,3.\\ 0, \text{otherwise}\end{cases}.$$

(a) Find the marginal PMF of X and Y

(b) Find the conditional PMF of $X|y = 2$

(c) Find $P\{X < Y\}$

Question .7 **(9+5=14-Points)** Let (X, Y) be a continuous RV with PDF

$$f(x,y) = \frac{\theta^{\alpha+r}}{\Gamma(\alpha)\Gamma(r)} x^{\alpha-1}(y-x)^{r-1}e^{-\theta y}, 0 < x < y < \infty$$ and o, otherwise, where $\alpha, r, \theta > 0$, and $\Gamma(.)$ is the gamma function.

(a) Find the marginal PDF of X

(b) Find the conditional PDF of $Y|X = x$

Question .8 **(7+8=15-Points)**

(a) Let X and Y be two independent RVs with PDFs $f(x) = \begin{cases} 1, 0 < x < 1 \\ 0, \text{otherwise} \end{cases}$, $f(y) = \begin{cases} \frac{1}{2\pi}, 0 < y < 2\pi \\ 0, \text{otherwise} \end{cases}$, find $P(X < Y)$

(b) Let X be an RV with MGF $M_x(t)$, for $-h < t < h$, $t > 0$. Show that

(i) $P\{X \geq a\} \leq e^{-at} M_x(t)$, for $0 < t < h$

(ii) $P\{X \leq a\} \leq e^{-at} M_x(t)$, for $-h < t < 0$