King Fahd University Of Petroleum & Minerals
Department Of Mathematics And Statistics
STAT501 : Probability and Mathematical Statistics (151)
First Exam **Monday October 12, 2015**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	12	
Two	8	
Three	18	
Four	7	
Five	8	
Six	14	
Seven	11	
Eight	12	
Total	90	

Question.1 (**4+4+4=12-Points**)

Let $\Omega = \{a, b, c, d\}$ be the space of a random experiment.

(a) Find the power set of Ω

(b) Let $\boldsymbol{A} = \{\{a\}\}$, find the σ-field generated by $\boldsymbol{A}$

(c) If $\boldsymbol{B} = \{\{a\}, \{a, b\}\}$, find the σ-field generated by $\boldsymbol{B}$

Question.2 (**5+3=8-Points**)

(a) Let $\mathfrak{F}$ be the class of all intervals of the form (x, ∞), $a \in \mathfrak{R}$. Is $\mathfrak{F}$ is a $\sigma-$filed? Explain.

(b) In a population of N members, there are F females, and $N - F$ males. If you select a sample of size $n(n < N)$ from this population, what is the probability that all members are males?

Question.3 (**10+8=18-Points**)

(a) Suppose that μ_1 and μ_2 are two measurable two functions on $(\Omega, \mathfrak{S})$. Show that the function $\mu = \mu_1 + \mu_2$ is also a measurable function on $(\Omega, \mathfrak{S})$.

(b) Show that if A and B are independent then A^c, B^c are also independent

Question.4 (**7-Points**)

Let $\Omega = \{0, 1, 2, \ldots, n\}$, n is positive integer, and $\mathfrak{S}$ be the $\sigma-$filed generated by Ω. Assume that for any $A \in \Omega$, $P(A) = \sum_{x \in A} \binom{n}{x} p^x (1-p)^{n-x}$, $x = 0, 1, \ldots, n$, $0 < p < 1$.Does $P(.)$ define a probability measure on $(\Omega, \mathfrak{S})$? Explain.

Question.5 (**8-Points**)

Let $(\Omega, \mathfrak{S}, P)$ be a probability space and let $A \in \mathfrak{S}$ with $P(A) > 0$, let $H \in \mathfrak{S}$, and $P_A(H) = P\{H|A\}$. Show that $(\Omega, \mathfrak{S}, P_H)$ is a probability space

Question .6 **(6+6+2=14-Points)**

Let $\{A_n\}, n \geq 1$ be the set of all points (x, y) that are given by:

$$A_n = \left\{(x, y) : 0 < x \leq 1 + \frac{1}{n}, \frac{1}{n+1} \leq y < 1\right\}$$

(a) Find, if exist, $\limsup A_n$

(b) Find, if exist, $\liminf A_n$

(c) Dose $\lim_{n \to \infty} A_n$ exists? Explain

Question .7 **(3+4+4=11-Points)**

An unbiased die is manufactured such that: two faces with number 0, two faces with number 1 and the last two with number 4 if the die is tossed two independent times. Then

(a) Write down the sample space Ω

(b) Let $\boldsymbol{X}$ be the number of times that 1 turns up,i.e. for every $\omega \in \Omega$, $\boldsymbol{X}(\omega) =$ Number of 1^s in ω. Show that $\boldsymbol{X}$ is a random variable.

(c) Find the distribution function of the random variable $\boldsymbol{X}$

Question .8 **(12-Points)**

Let $F(x) = \frac{1}{\pi}\tan^{-1}(x) + \frac{1}{2}, \quad -\infty < x < \infty$. Show that $F(x)$ defines a distribution function (DF) of the random variable $\boldsymbol{X}$