King Fahd University Of Petroleum & Minerals Department of Mathematics & Statistics STAT416 : Stochastic Processes for Actuaries (151) Assignment # 1 (Due Wednesday October 28, 2015)

- Q.1 Let X_n be the weather on day n in Al-Dhahran city, with state 0 = Sunny and state 1 = rainy. If the wetaher today is sunny then it will be tomorrow sunny with probability 0.90, while this probability decreases if the current weather today is rainy to 0.70. Then answer the following
 - a. Why the weather case represents a Markov Chain?
 - b. Write the probability transition matrix for this Markov Chain?
 - c. Suppose that the weather today(Sunday) is sunny, what is the probability that it will be sunny on Tuesday?
- Q.2 A Markov chain with three states 0, 1, 2 and has the following transition probability matrix

$$\boldsymbol{P} = \left(\begin{array}{rrr} 0.7 & 0.2 & 0.1 \\ 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \end{array}\right).$$
 Then

- (a.) Are the three states communicate with each othe? Expalain.
- (b.) Find the classes and the period of this Markov chain
- (c.) Is the Markov chain in this question irreduicable?Explain
- (d.) Find $P(X_3 = 1, X_2 = 1 | X_1 = 0)$
- Q.4 A soccer player either makes a penalty hit goal or fault. Assume that if he makes a goal, the probability that the next hit is a fault is 0.40. If the first hit is fault, then he pays more attention and the probability that the next is fault is 0.20. Denote a goal by state 0 and a fault by state 1.
 - a. Find the transition probability matrix .
 - b. what is the probability that the fifth hit is a goal, given that the second hit was goal?(6-Points)
- $\mathbf{Q.5}\,$ Solve problem 14 page 277 of your text book.
- Q.6 Solve problem 66 page 286 of your text book