KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
STAT416 : Stochastic Processes for Actuaries (151)
Third Exam **Wednesday December 2, 2015**

Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	12	
Two	12	
Three	15	
Four	7	
Five	10	
Six	18	
Seven	14	
Eight	12	
Total	100	

Question.1 (**2+2+2+2+4=12-Points**)

Explain the meaning or define the following:

(a) If $\{X(t); t \geq 0\}$ denotes to the number of customers in a queueing system at time t, and $P_3 = 0.25$, what is the meaning of this number?

(b) Explain the PASTA property.

(c) In the queueing system $A_1/A_2/A_3/A_4/A_5$ what is the meaning of A_4?

(d) What is the meaning of stationary increments for a Brownian motion $\{X(t); t \geq 0\}$?

(e) For an $M/M/3$ queueing system we get $E(L_q) = 1.5$ and $E(W) = \frac{1}{2}$. Explain the meaning of these two numbers.

Question.2 **(3+3+3+3=12-Points)**

Customers arrive at a checkout counter in a grocery store according to a Poisson process with an average rate of 10 customers per hour. There is only one clerk at the counter, and the time to serve each customer is exponentially distributed with mean of 4 minutes.

(a) Specify the type of queueing system in this grocery store.

(b) Find the probability that a queue forms at the counter. (i.e: the clerk is busy).

(c) Find the average time a customer spends at the counter (i.e: The system)

(d) Find the average queue length at the counter?

Question.3 **(3+7+5=15-Points)**

Students arrive at a checkout counter in the university coffee shop according to a Poisson process with an average of 15 students per hour. There are two cashiers at the counter, and they provide identical service to the students. The time to serve a student by either cashier is exponentially distributed with mean of 3 minutes. Students find both cashiers busy on their arrival join a single queue.

(a) Specify the type of queueing system and find the corresponding parameters

(b) Find the limiting probabilities for this queueing system

(c) what is the probability that an arriving student does not need to wait?

Question.4 **(7-Points)**

Derive the limiting probabilities for an $M/M/1/k$ queueing system.

Question 5. **(6+4=10-Points)**

Suppose that each day people arrive at Adam's garage to have their cars fixed. Adam's garage can only accommodate four cars. Anyone arriving when there are four cars in the garage has to go a way without leaving his car for Mr. Adam to fix. Assume that the people arrive according to a Poisson process at rate of one customer per hour, and the time it takes Adam to service a car is exponentially distributed with a mean of 45 minutes.

(a) What is the probability that a customer arrive finds Mr. Adam's garge empty?

(b) What is the probability that an arriving customer leaves without getting his car fixed?

Question 6. (**11+4+3=18-Points**)

Consider a network of three stations with a single server at each station. Customers arrive at stations 1, 2, 3 in accordance with Poisson processes having respective rates 5, 10, and 15. The service times at the three stations are exponential with respective rates 10, 50, and 100. A customer completing service at station 1 is equally likely to (i) go to station 2, (ii) go to station 3, or (iii) leave the system ($\pi_{12} = \pi_{13} = \frac{1}{3}, \pi_{11} = 0$). A customer departing service at station 2 always goes to station 3 ($\pi_{23} = 1$). A departure from service at station 3 is equally likely to either go to station 2 or leave the system ($\pi_{32} = \frac{1}{2}, \pi_{33} = 0$).

(a) What is the limiting probability of having one customer at each station?

(b) What is the average number of customers in the system (consisting of all three stations)?

(c) What is the average time a customer spends in the system?

Question 7. (**6+5+3=14-Points**)

Let $\{X(t); t \geq 0\}$ be a Brownian motion process with variance $\sigma^2 = 4$. Find the following:

(a) $P(X(2) \leq 2)$

(b) Find $var(2X(4) - 3X(5))$

(c) Find the distribution of $X(3) + X(1)$

Question 8. (**5+7=12-Points**)

The price of a stock follows a standard Brownian motion process $\{X(t); t \geq 0\}$. Given that the price at time $t = 2$ is $\frac{1}{4}$

(a) What is the probability that the stock price at $t = 3$ will be less than $\frac{1}{2}$

(b) Find $P\left(X(4) - X(2) > \frac{1}{2} | X(2) = \frac{1}{2}\right)$