KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 151
STAT416 : Stochastic Processes for Actuaries (151)
Second Exam **Wednesday November 4, 2015**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	18	
Two	20	
Three	16	
Four	12	
Five	14	
Six	10	
Total	90	

Some Formulas:

(1.) $\displaystyle\sum_{j=0}^{k} r^j = \frac{1-r^{k+1}}{1-r}$, $r \neq 1$

(2.) $\displaystyle\sum_{j=m}^{\infty} r^j = \frac{r^m}{1-r}$, $|r| < 1$

(3.) $\displaystyle P\{S_n^1 < S_m^2\} = \sum_{k=n}^{n+m-1} \binom{n+m-1}{k} \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k \left(1 - \frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{n+m-1-k}$

Question.1 (**2+8+6+2=18-Points**)

Consider a branching process where the offsprings distribution is given by:

$$b_k = \frac{1}{3}\left(\frac{2}{3}\right)^k, \quad k = 0, 1, 2, \ldots$$

(a) Write down the probability generating function $\beta(z)$.

(b) Find $P\{X_2 = 0 | X_0 = 1\}$

(c) Find the probability of extinction (π_0) given that the process starts with one individual only. (Hint: solve $z = \beta(z)$)

(d) Find $P\{X_1 = 4 | X_0 = 1\}$

Question.2 **(6+7+3+4=20-Points)**

Patients arrive at the doctors's office according to a Poisson process with rate (1) patient per (10) minutes. Assume that the doctor will not start working until at least three patients are in the waiting room.

(a) Find the probability that no patients is admitted to see the doctor during the first hour.

(b) Find the probability that exactly one patient arrive during the first 20 minutes and six patients during the first hour.

(c) Find the expected waiting tome until the first patient is admitted to to see the doctor

(d) Find the probability that the time between the seventh and the eighth patient excess (20) minutes.

Question 3. **(4+6+2+4=16-Points)**

A mall has to major entrances, one form street A and the other from street B. Customers arrive to the mall using these two streets according to a Poisson process $\{N(t), t \geq 0\}$ with rate $\frac{1}{2}$ per minute from street A, $\frac{3}{2}$ per minute from street B. Given these information, answer the following:

(a) What is the probability that no customers enter the mall during a period of 3 minutes?

(b) What is the probability that fewer that two customers will arrive the mall from street A in a given minute?

(c) What is the expected number of customers who will enter them mall using street B na period of one hour?

(d) Find $P\{N(3) = 4|N(7) = 5\}$

Question 4. **(12-Points)**

Suppose that a one cell organism can be in one of 3 states A or B or C. An individual in state A will change to state B at an exponential rate α. An individual in state B will change to state A at an exponential rate β, and to state C at an exponential rate β. An individual in state C will change to state B at exponential rate γ. For the continuous time Markov process $\{X(t)\}$ for a population of such organisms, determine the following: P_{AA}, P_{AB}, P_{BA}, P_{BB}, P_{BC}, P_{CA}, P_{CB}, P_{CC}, ν_A, ν_B, q_{AB}, q_{BA}.

Question 5. **(4+10=14-Points)**

Consider a continuous Markov chain $\{X(t), t \geq 0\}$ with only two states $\{0, 1\}$. Assume that the two states are not absorbing, and the rate from 0 to 1 is λ, and from 1 to 0 is μ (a birth and death process).

(a) Write down the four Backward Kolomogrov differential equations.

(b) If $\lambda = 3$, $\mu = 2$, solve for $P_{00}(t)$ to calculate $P_{00}(1)$.

Question 6. **(10-Points)**

Given the rate matrix $\boldsymbol{Q} = \begin{pmatrix} -2 & 2 & 0 & 0 \\ 2 & -4 & 2 & 0 \\ 2 & 0 & -4 & 2 \\ 2 & 0 & 0 & -2 \end{pmatrix}$. Find the limiting probability vector $\boldsymbol{\pi}$.