King Fahd University Of Petroleum & Minerals
Department Of Mathematics And Statistics
STAT416 : Stochastic Processes for Actuaries (151)
First Exam **Wednesday October 7, 2015**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	10	
Two	12	
Three	8	
Four	15	
Five	10	
Six	15	
Total	70	

Question.1 (**2+2+2+2+2=10-Points**) Define the following:

(a) Markov Chain:

(b) Communicate states: (say i and j)

(c) Recurrent state:

(d) Period of a state:

(e) An absorbing state:

Question .2 **(2+3+7=12-Points)**

Suppose that we have a Markov chain with three states {0,1,2}. If the process starts in state 0 then after $n-$steps it will be in state $0, 1, 2$ with probabilities $0.15, 0.25, 0.60$, respectively. If it starts in state 1 then after $n-$steps it will be in state $0, 1, 2$ with probabilities $0.45, 0.30, 0.625$, respectively, and if it starts in state 2 then after $n-$steps it will be in state $0, 1, 2$ with probabilities $0.35, 0.20, 0.45$, respectively. Let $\{X_n\}$ be the state is in state 0 or 1 or 2 after $n-$steps.

(a) Why this is a Markov chain? Explain.

(b) Write the transition probabilities matrix $\boldsymbol{P}$.

(c) Find the probability that the process is in state 2 on step 6, in state 1 on step 5 and in state 0 on step 4 given that it was in state 2 on step 3.

Question.3 **(4+4=8-Points)**

Consider the following Markov chain with states {0,1,2,3} and transition probabilities matrix given by

$$\boldsymbol{P} = \begin{pmatrix} 0.5 & 0 & 0.5 & 0 & 0 \\ 0.25 & 0.5 & 0.25 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0.5 \end{pmatrix}. \text{ Then}$$

(a.) Determine the classes,and specify which are recurrent or transient states

(b.) Find the the period of the recurrent states. Is it periodic or aperiodic? Why?

Question.4 **(6+9=15-Points)**

Suppose that we have a Markov chain with two states {0,1} and

$$\boldsymbol{P}^n = \frac{1}{13}\begin{pmatrix} 7+6(-0.3)^n & 6-6(-0.3)^n \\ 7-7(-0.3)^n & 6+7(-0.3)^n \end{pmatrix}$$

Given that $P\{X_0 = 0\} = 0.40$.

(a) If the intial vector is $(0,1)$ find $\lim_{n\to\infty} \boldsymbol{P}^n$

(b) Find $E(X_3)$

Question 5. **(8+2=10-Points)**

A Markov chain with states {0,1,2,3} has the following transition probabilities matrix

$$\boldsymbol{P} = \begin{pmatrix} 0.50 & 0.50 & 0 & 0 \\ 0.25 & 0.50 & 0.25 & 0 \\ 0 & 0.25 & 0.50 & 0.25 \\ 0 & 0 & 0.50 & 0.50 \end{pmatrix}$$

(a) Find the limiting probability vector ($\boldsymbol{\pi}$)

(b) Find the long-run expected value of the process of this Markov chain.

Question 6. (**12+3=15-Points**)

A gambler makes a series of bets \$1 starts with \$1. If the probability of wining each bet is 0.50. He will quit when he wins \$25 or losses \$50. (Note: A loss means negative value)

(a) Find the probability he quits with \$25.

(b) What is the probability the gambler losses \$25 during this game?