**Q.1: (6+1 = 7 points)**

In the air pollution study performed at an experiment station, it is claimed that the true mean amount of suspended benzene-soluble organic matter is not less than 2.5 μg/m³.
The following amount of suspended benzene-soluble organic matter (in micrograms per cubic meter) were obtained for eight different samples of air. These measurements (X) are given below:
X: 2.2, 1.8, 3.1, 2.0, 2.4, 2.0, 2.1, 1.2
The descriptive measures for the dataset are: mean=2.1, standard deviation=0.54

Use this information to answer the following questions.

a. Using the p-value approach, test the claim at a 2.5% level of significance. (State your hypotheses and show other necessary steps of testing).

$n = 8$ ; $\bar{x} = 2.1$ ; $s = 0.54$

$H_0: \mu \geq 2.5$ $\quad H_1: \mu < 2.5$ (1)

$$T_c = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.1 - 2.5}{0.54/\sqrt{8}} = -2.095 \text{ (1)}$$

$p\text{-value} = P[T_7 < -2.095] = P[T_7 > 2.095]$ (1)

From the table

$0.035 < p\text{-value} < 0.05$ (1)

Since $(p\text{-value}) \nless (\alpha = 0.025)$ so we fail to (1) reject $H_0$ and conclude true mean (1) amount of . . . . . . is not less than 2.5

b. Write the assumption(s) if (any) to test your hypotheses for part (a) above.

Normality assumption is needed. (1)

**Q.2: (7+2 = 9 points)**

It is claimed that the resistance of electric wire can be reduced by more than 0.05ohm by alloying. To test the claim, 32 values obtained for standard wire had a mean 0.136 ohm and a standard deviation 0.004 ohm, and 17 values obtained for alloyed wire had a mean 0.083 ohm and a standard deviation 0.005 ohm.

a. Do the data support the claim? Use critical value approach to test the said claim at 5% level of significance. (State your hypotheses and show other necessary steps of testing)

$n_1 = 32$ ; $\bar{x}_1 = 0.136$ ; $s_1 = 0.004$

$n_2 = 17$ ; $\bar{x}_2 = 0.083$ ; $s_2 = 0.005$

$H_0: \mu_1 - \mu_2 \le 0.05$ $\quad H_1: \mu_1 - \mu_2 > 0.05$ 1

$$T_c = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(0.136 - 0.083) - 0}{0.0044\sqrt{\frac{1}{32} + \frac{1}{17}}}$$

$T_c = 2.289$ 1

$T_{0.05,47} = \frac{1.6772 + 1.6787}{2} = 1.67795$ 1

1 Assuming equal variances

$$s_p^2 = \frac{31(0.004)^2 + 16(0.005)^2}{47} = 1.9 \times 10^{-5}$$

$s_p = 0.0044$ 1

As $(T_c = 2.289) > (T_{0.05,47} = 1.678)$ 1 So we reject $H_0$ and 1 conclude That resistance can be reduced by more Than $0.05\,\Omega$

b. Write the assumption(s) if (any) to test your hypotheses for part (a) above.

- Normality
- Equal variances
- Independent Samples

Any TWO

1 1

**Q.3:- (4+1+4+3 = 12 points)**

A random sample of 50 suspension helmets used by motorcycle riders and automobile race-car drivers was subjected to an impact test, and on 18 of these helmets some damage was observed.

a. Find a 95% two-sided confidence interval on the true proportion of helmets of this type that would show damage from this test.

$n = 50$, $x = 18$, $\hat{p} = 0.36$ (1)

$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ (2)

$0.36 \pm 1.96\sqrt{\frac{0.36(0.74)}{50}}$

$(0.23, 0.49)$ (1)

b. Interpret the interval you obtained in part (a)

We are 95% confident that the true proportion will lie b/w (0.23, 0.49) (1)

c. Using the point estimate of $p$ obtained from the preliminary sample of 50 helmets, how many helmets must be tested to be 95% confident that the error in estimating the true value of $p$ is less than 0.02?

$\hat{p} \simeq 0.36$ 1

$$n \geq \left(\frac{z_{\alpha/2}}{e}\right)^2 \hat{p}(1-\hat{p}) = \left(\frac{1.96}{0.02}\right)^2 (0.36)(0.64)$$

1 1

$$= 2212.762 \simeq 2213$$

1

d. How large must the sample be if we wish to be at least 95% confident that the error in estimating $p$ is less than 0.02, regardless of the true value of $p$?

$\hat{p} = \frac{1}{2}$ 1

1

$$n \geq \left(\frac{z_{\alpha/2}}{e}\right)^2 \left(\frac{1}{4}\right) = \left(\frac{1.96}{0.02}\right)^2 \frac{1}{4} = 2401$$

1

**Q.4:- (3+4=7points)**

a) Following are two confidence interval estimates of the mean μ of the cycles to failure of an automotive door latch mechanism (the test was conducted at an elevated stress level to accelerate the failure).

CI 1: $3124.9 \leq \mu \leq 3215.7$ CI 2: $3110.5 \leq \mu \leq 3230.1$

The confidence level for one of these CIs is 95% and the confidence level for the other is 99%. Both CIs are calculated from the same sample data. Which is the 95% CI? Explain why.

Length of CI 1 = 90.8
Length of CI 2 = 119.6 — 1

95% CI is the one with shorter length 2

So CI 1 is 95%.

b). A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed with standard deviation 2 volts, and the manufacturer wishes to test $H_0$: $\mu \geq \mu_0$ against $H_1$: $\mu < \mu_0$. A sample of size 16 is selected and the sample mean is found to be 20.41 volts. After testing the above mentioned hypotheses, the manufacturer got p-value = 0.119. Find the value of $\mu_0$.

Given $\sigma = 2$, $n = 16$, $\bar{x} = 20.41$

$H_0: \mu \geq \mu_0$, $H_1: \mu < \mu_0$

p-value = 0.119

$\Rightarrow Pr(Z < Z_0) = 0.119$ — 1

$\Rightarrow Pr\left(Z < \frac{\bar{x} - \mu_0}{\sigma_0/\sqrt{n}}\right) = 0.119$

From Z Table 1
$P_r(Z < -1.18) = 0.119$

$\Rightarrow \frac{\bar{x} - \mu_0}{\sigma_0/\sqrt{n}} = -1.18$ — 1

$\Rightarrow \frac{20.41 - \mu_0}{2/\sqrt{16}} = -1.18 \Rightarrow \mu_0 = 1.18\left(\frac{1}{2}\right) + 20.41$
$= 21$ — 1