KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 319: Probability & Statistics for Engineers & Scientists
Semester 151
Second Major Exam
Wednesday October 28, 2015
5:10 – 6:40 pm

Please encircle your instructor name:

Abbas Al-Sawi Anabosi

Malik Riaz Samouh

Name: KEY ID #: Section #: Serial #:

Question No	Full Marks	Marks Obtained
1	07	
2	07	
3	09	
4	09	
5	08	
Total	40	

Q.No.1:- (4+3 = 7 points) The density function of the random variable X, the total number of hours (in units of 100 hours) that a family runs a vacuum cleaner over a period of one year, is

$$f(x) = \begin{cases} \frac{1}{4}(x^2 + x + 1), & 0 < x < 1 \\ \frac{13}{80}(6 - x^2), & 1 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

a) Is this a valid probability density function? How? Justify your answer.

$$= \int_0^1 \frac{1}{4}(x^2+x+1)\,dx + \int_1^3 \frac{13}{80}(6-x^2)\,dx \quad (0.5)$$

$$= \frac{1}{4}\left|\frac{x^3}{3}+\frac{x^2}{2}+x\right|_0^1 + \frac{13}{80}\left|6x - \frac{x^3}{3}\right|_1^3$$

$$= \frac{1}{4}\left(\frac{1}{3}+\frac{1}{2}+1\right) + \frac{13}{80}\left(18-9-6+\frac{1}{3}\right) \quad (1)$$

$$= \frac{1}{4}\left(\frac{2+3+6}{6}\right) + \frac{13}{80}\left(3+\tfrac{1}{3}\right)$$

$$= \frac{1}{4}\times\frac{11}{6} + \frac{13}{80}\times\frac{10}{3} = \frac{11}{24}+\frac{13}{24} = \frac{24}{24} = 1 \quad (1)$$

No (0.5)
Because $f(x)$ is negative for $x > 2.5$.
(2.5)

b) Find the average number of hours per year that families run their vacuum cleaners.

$$\mu = \int_0^1 x \cdot \frac{1}{4}(x^2+x+1)\,dx + \int_1^3 \frac{13}{80}x(6-x^2)\,dx \quad (2)$$

$$= \frac{1}{4}\int_0^1 (x^3+x^2+x)\,dx + \frac{13}{80}\int_1^3 (6x-x^3)\,dx$$

$$= \frac{1}{4}\left(\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}\Big|_0^1\right) + \frac{13}{80}\left(\frac{6x^2}{2}-\frac{x^4}{4}\Big|_1^3\right) \quad (1)$$

$$= \frac{1}{4}\left(\frac{1}{4}+\frac{1}{3}+\frac{1}{2}\right) + \frac{13}{80}\left(3(9)-\frac{81}{4}-3+\frac{1}{4}\right)$$

$$= \frac{1}{4}\left(\frac{3+4+6}{12}\right) + \frac{13}{80}\left(24-\frac{80}{4}\right)$$

0.271 + 0.65

$$= \frac{13}{48} + \frac{13}{80}(4) = \frac{13}{48}+\frac{13}{20} = 0.921 \quad (1.5)$$

(04)
Because $f(x)$ is not a pdf. So we cannot find $E(X)$

Q.No.2:- (3+4 = 7 points) The length of the tube rods (in cm) is normally distributed with mean 80 cm and standard deviation 0.5 cm. If the length of the tube rod is between 79.4 cm and 80.8 cm, it is considered as non-defective. Otherwise, the rod is declared defective.

a) What is the probability that a randomly selected tube rod is defective?

$P = P(79.4 < x < 80.8)$ (1)

$= P(-1.2 < Z < 1.6)$

$= \phi(1.6) - \phi(-1.2)$

$= 0.9452 - 0.1151$ (1)

$= 0.8301$ (Non defective)

$= 1 - 0.8301$

$= 0.1699$ (Defective (1)

b) If we have a lot of 2500 tube rods, what is the probability that at least 2100 are non-defective?

$n = 2500$
$P = 0.8301$ (1)

$nP = 2075.25$
$npq = 352.585$ (1)

$P(x \geq 2100)$ (1)

$P\left(z \geq \frac{2099.5 - 2075.25}{\sqrt{352.585}}\right)$

$P(Z \geq 1.29) = 0.0985$ (1)

$1 - P(Z < 1.29) = 1 - 0.9015 = 0.0985$

$= 0.0985$

Q.No.3:- (2+3+4 = 9 points) The net weight in pounds of a packaged chemical herbicide is uniform for $49.75 < x < 50.25$ pounds.

a) Find the probability that a randomly selected package has a weight between 50 to 50.4 pounds.

$f(x) = \frac{1}{0.5} = 2$

$P(50 < x < 50.4) = \int_{50}^{50.4} 2\,dx = 2\int_{50}^{50.25} dx + \int_{50.25}^{50.4} 0\,dx$ (1)

$= 2\left(x\Big|_{50}^{50.25}\right) = 2(50.25 - 50)$

$= 2(0.25) = 0.5$ (1)

b) Find the mean, variance and median of the weight of packages.

$\mu = \frac{a+b}{2} = \frac{49.75 + 50.25}{2} = 50$ (1)

$\sigma^2 = \frac{(b-a)^2}{12} = \frac{(0.5)^2}{12} = 0.0208$ (1)

median = mean = 50. (1)

c) If a random sample of size 35 packages is selected, what is the probability that the absolute difference between the sample mean and population mean is less than 0.005 pounds?

$= P(|\bar{x} - \mu| < 0.005)$ (1)

$= P\left(|Z| < \frac{0.005}{\sqrt{\frac{0.0208}{35}}}\right)$ (1)

$= P(|Z| < 0.205)$

$= 1 - 2 \times 0.4168$ (1)

$= 0.1664$ (1)

OR

$= P(-0.205 < Z < 0.205)$

$= P(Z < 0.21) - P(Z < -0.21)$

$= 0.5832 - 0.4168$

$= 0.1664$

Q.No.4:- (3+3+3 = 9 points) A computer time-sharing system receives telephone inquiries according to Poisson distribution at an average rate of 0.1 per minute.

a) What is the probability that there are no telephone inquiries in first 10 minutes of the day?

Method 1 $t = 10 \quad \lambda t = 10 \times 0.1 = 1$ (1)

$$P(X = x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} \quad (1)$$

$$P(X=0) = \frac{1^0 e^{-1}}{0!} = e^{-1} = 0.3679 \quad (1)$$

Method 2.

$$P(Y > 10) = \int_{10}^{\infty} 0.1\, e^{-0.1y}\, dy$$

$$= -e^{-0.1y}\Big|_{10}^{\infty}$$

$$= 0 + e^{-1}$$

$$= 0.3679$$

b) What is the probability that the first telephone inquiry is received in less than 1 minute?

$$P(y < 1) = \int_0^1 0.1\, e^{-0.1y}\, dy = -e^{-0.1y}\Big|_0^1 = -e^{-0.1} + e^0$$

(1) (1)

$$= 1 - e^{-0.1}$$

$$(1) = 0.095$$

c) If there is no telephone inquiry received in first 10 minutes of the day, then what is the probability that there are no telephone inquiries in the next 5 minutes?

Method 1 $\lambda = 0.1 \quad t = 5 \quad \lambda t = 1/2$ (1)

$$P(X = x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$$

Memory less (1)

$$P(X=0) = \frac{(1/2)^0}{0!} e^{-1/2} = 0.6065 \quad (1)$$

Method 2.

$$P(Y > 5) = \int_5^{\infty} 0.1\, e^{-0.1y}\, dy.$$

$$= -e^{-0.1y}\Big|_5^{\infty}$$

$$= 0 + e^{-1/2}$$

$$= 0.6065$$ A

Q.No.5:- (3+3+2 = 8 points) The accompanying data show the single-leg power at a high workload.

214	191	160	207	180	176	174	205
211	183	252	189	191	221	230	

For the above data, answer the following in detail:

a) Compute the coefficient of skewness and comment on its value.

$\bar{x} = 198.93$ (1/2) $\quad \tilde{x} = \text{Median} = 191$ (1/2)

$s = 24.265$ (1)

$CS = \frac{3(\bar{x}-\tilde{x})}{s} = \frac{3(198.93-191)}{24.265} = 0.981$ (1/2)

Skewed towards right or Positively skewed (1/2)

b) Construct a boxplot and comment on the shape, showing outliers (if any!).

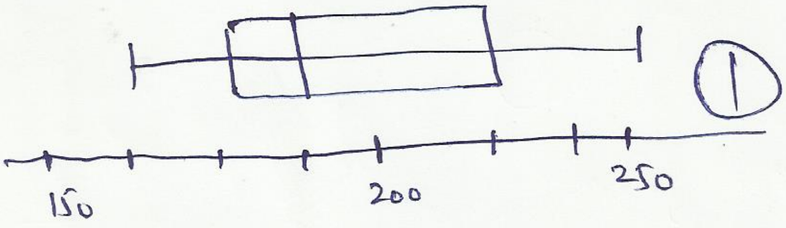
Min = 160 $\quad Q_1 = 180 \quad$ Median = 191 (1)

$Q_3 = 214 \quad$ Max = 252 $\quad$ IQR = 34

$Q_1 - 1.5\,IQR = 129$

$Q_3 + 1.5\,IQR = 265$

So no outlier (1)



(1)

c) Using the z-score, do you think that 252 is an outlier?

The z-score for 252 is 2.187. (1)

So 252 is not an outlier. (1)