STAT 319
maj 1. 151
Solution Key

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 319: Probability & Statistics for Engineers & Scientists
Semester 151
First Major Exam –
Wednesday September 16, 2015
6:00 – 7:15 pm

Please encircle your instructor name:

Abbas   Al-Sawi   Anabosi
Malik   Riaz   Samouh

Name: ID #:   Section #:   Serial #:

Question No	Full Marks	Marks Obtained
1	**8**	
2	**8**	
3	**7**	
4	**6**	
5	**6**	
Total	**35**	

Q.No.1 (3+3+2=8 points):- The number of arrivals at a local gas station between 3:00 and 5:00 P.M. has a Poisson distribution with a mean of 12.

a. Find the probability that the number of arrivals between 3:00 and 5:00 P.M. is at least 1.

$\lambda = 12$ per 2 hours

$X \sim Poisson(12) \Rightarrow P(X = x) = \frac{12^x e^{-12}}{x!}, \quad x = 0, 1, 2, \ldots$ (1)

$Pr(X \geq 1) = 1 - Pr(X = 0)$ (1)

$= 1 - \frac{12^0 e^{-12}}{0!}$

$= 1 - e^{-12}$

$= 1 - \cancel{0.1353}$ 0.0000

$= \cancel{0.8647}$ 1.0000 (1)

b. Find the probability that the number of arrivals between 3:30 and 4:00 P.M. is at most 1.

$\lambda = 12, \quad t = \frac{1}{4} \Rightarrow \lambda t = \cancel{\frac{1}{4}}\ 3$ (1)

$P(X = x) = \frac{(\frac{3}{2})^x e^{-\frac{3}{2}}}{x!}, \quad x = 0, 1, 2, \ldots$

$Pr(X \leq 1) = P(X = 0) + P(X = 1)$ (1)

$= \frac{(\frac{3}{2})^0 e^{-\frac{3}{2}}}{0!} + \frac{(\frac{3}{2})^1 e^{-\frac{3}{2}}}{1!}$

$= e^{-\frac{3}{2}}(1 + \frac{1}{2}) = \frac{3}{2} e^{-\frac{3}{2}} = 0.20\cancel{08}$ (1)

c. Find variance for the number of arrivals between 4:00 and 5:00 P.M.

$\lambda = 12, \quad t = \frac{1}{2} \Rightarrow \lambda t = 6$ (1)

so variance $= 6$ (1)

Q.No.3 (3+3+2=8 points):- Suppose that of all individuals buying a certain personal computer, 60% include a word processing program in their purchase, 40% include a spreadsheet program, and 30% include both types of programs. Consider randomly selecting a purchaser and let A = (word processing program included) and B = (spreadsheet program included).

a. Find the probability that a word processing program or a spread sheet program was included.

$P(A) = 0.6, \quad P(B) = 0.4, \quad P(A \cap B) = 0.3$ 1

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ 1

$= 0.6 + 0.4 - 0.3 = 0.7$ 1

b. Find the probability that a word processing program was included given that the selected individual included a spreadsheet program.

1 $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.4} = \frac{3}{4} = 0.75$ 1

1

c. Are A and B independent? How? Justify your answer.

1 NO,

because $P(A/B) \neq P(A)$ 1

$0.75 \neq 0.6$ 1

or $P(A \cap B) \neq P(A)\,P(B)$

$0.3 \neq 0.6 \times 0.4 \; (= 0.24)$

Q.No.3 (2+5=7 points):- A company has 2 machines that produce widgets. An older machine produces 23% defective widgets, while the new machine produces only 8% defective widgets. In addition, the new machine produces 3 times as many widgets as the older machine does.

a. Given that a widget was produced by the new machine, what is the probability it is not defective?

O $\frac{1}{4}$ — D/O .23 $0.23 \times 0.25 = 0.0575$

N $\frac{3}{4}$ — D/N 0.08 → 1 $0.08 \times 0.75 = 0.06$

$\Rightarrow P(\bar{D}/N) = 0.92$ → 1

$P(D) = 0.1175 \Rightarrow P(\bar{D}) = 1 - 0.1175$

$= 0.8825$

b. Given that a widget is not defective, what is the probability it was produced by the new machine?

O $\frac{1}{4}$ — $\bar{D}/O$ 0.77 $0.77 \times 0.25 = 0.1925$ 1

N $\frac{3}{4}$ — $\bar{D}/N$ 0.92 $0.92 \times 0.75 = 0.69$ 1

$P(\bar{D})$ 0.8825 1

$$P(N/\bar{D}) = \frac{P(N \cap \bar{D})}{P(\bar{D})} \quad 1$$

$$= \frac{0.69}{0.8825}$$

$$= 0.78187 \quad 1$$

Q.No.4 (3+3=6 points):- A day's production of 12 manufactured parts contains 3 parts that do not meet customer requirements. Three parts are selected randomly without replacement from the batch.

a. Find the probability that the first part is not defective and the 2nd and 3rd are defective.

D	$\bar{D}$	T
3	9	12

$$P(\bar{D}\cap D\cap D) = \frac{9}{12} \times \frac{3}{11} \times \frac{2}{10}$$

1 1 1

b. Find the probability that any two (out of three selected) parts are defective. 1

$$P(2 \text{ Defectives}) = 1 \; \frac{{}^{3}C_{2}\;{}^{9}C_{1}}{{}^{12}C_{3}} = \frac{3 \times 9 \times \cancel{6}}{\cancel{12}_{2} \times 11 \times 10} = \frac{27}{220}$$

1

$= 0.1228$

Q.No.5 (3+3=6 points):- The probability that a patient recovers from a delicate heart operation is 0.8. For the next three patients who have this operation:

a. What is the probability that exactly 2 patients survive?

$n = 3, \quad p = 0.8$

1

X = Number of patients that recover from operation

$X \sim Bin(3, 0.8)$

$P(X = 2) = {}^{3}C_{2}\ 0.8^{2}\ 0.2^{1}$ 1

$= 3(0.64)(0.2)$

$= 0.384$ 1

b. What is the average number of survived patients?

$\mu = np = 3(0.8) = 2.4$

1 1 1