

STAT 319
May 1, 151
Solution Key

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 319: Probability & Statistics for Engineers & Scientists
Semester 151
First Major Exam - 8
Wednesday September 16, 2015
6:00 - 7:15 pm

Please encircle your instructor name:

- Abbas
- Al-Sawi
- Anabosi
- Malik
- Riaz
- Samouh

Name: _____ ID #: _____ Section #: _____ Serial #: _____

Question No	Full Marks	Marks Obtained
1	8	
2	8	
3	7	
4	6	
5	6	
Total	35	

Q.No.1 (3+3+2=8 points):- The number of arrivals at a local gas station between 3:00 and 5:00 P.M. has a Poisson distribution with a mean of 12.

a. Find the probability that the number of arrivals between 3:00 and 5:00 P.M. is at least 1.

$$\lambda = 12 \text{ per 2 hours}$$

$$X \sim \text{Poisson}(\lambda) \Rightarrow P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x=0, 1, 2, \dots \quad (1)$$

$$Pr(X \geq 1) = 1 - P(X=0) \quad (1)$$

$$= 1 - \frac{12^0 e^{-12}}{0!}$$

$$= 1 - e^{-12}$$

$$= 1 - 0.0000053$$

$$= 0.9999947 \quad (1)$$

b. Find the probability that the number of arrivals between 3:30 and 4:00 P.M. is at most 1.

$$\lambda = 12, \quad t = \frac{1}{4} \Rightarrow \lambda t = \frac{12}{4} = 3 \quad (1)$$

$$P(X=x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}, \quad x=0, 1, 2, \dots$$

$$Pr(X \leq 1) = P(X=0) + P(X=1) \quad (1)$$

$$= \frac{(\lambda t)^0 e^{-\lambda t}}{0!} + \frac{(\lambda t)^1 e^{-\lambda t}}{1!}$$

$$= e^{-3} \left(1 + \frac{1}{2} \right) = \frac{3}{2} e^{-3} = 0.20 \quad (1)$$

c. Find variance for the number of arrivals between 4:00 and 5:00 P.M.

$$\lambda = 12, \quad t = \frac{1}{2} \Rightarrow \lambda t = 6 \quad (1)$$

$$\text{So Variance} = 6 \quad (1)$$

Q.No.3 (3+3+2=8 points):- Suppose that of all individuals buying a certain personal computer, 60% include a word processing program in their purchase, 40% include a spreadsheet program, and 30% include both types of programs. Consider randomly selecting a purchaser and let A = (word processing program included) and B = (spreadsheet program included).

- a. Find the probability that a word processing program or a spreadsheet program was included.

$$P(A) = 0.6, \quad P(B) = 0.4, \quad P(A \cap B) = 0.3 \quad (6)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (1)$$

$$= 0.6 + 0.4 - 0.3 = 0.7 \quad (1)$$

- b. Find the probability that a word processing program was included given that the selected individual included a spreadsheet program.

$$(1) P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.4} = \frac{3}{4} = 0.75 \quad (1)$$

- c. Are A and B independent? How? Justify your answer.

(1) NO,
because $P(A|B) \neq P(A)$ (6)

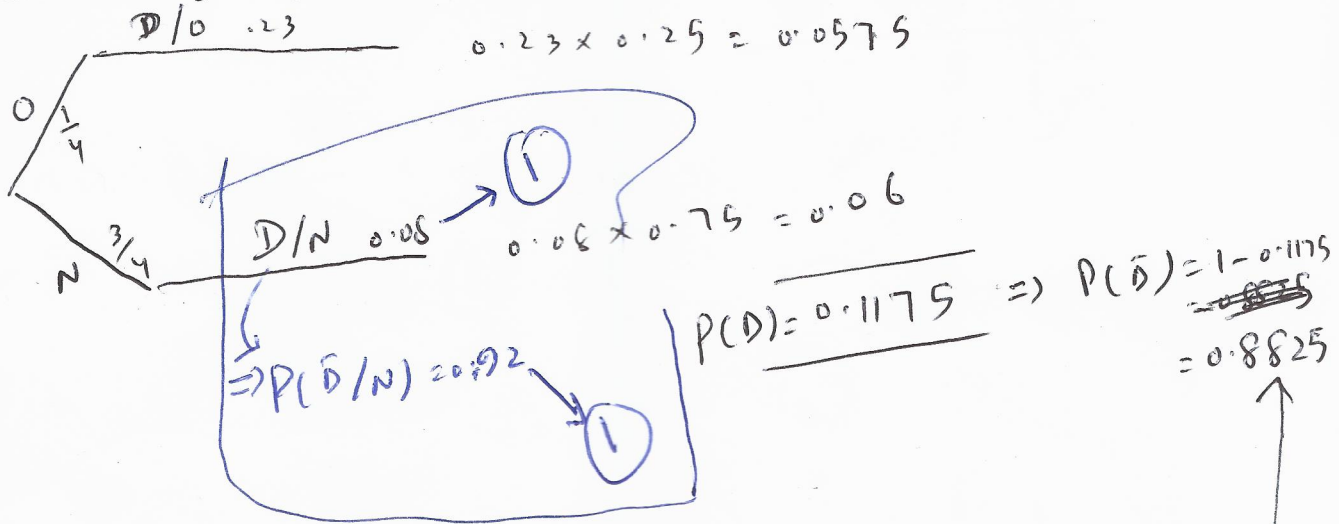
$$0.75 \neq 0.6 \quad (1)$$

or $P(A \cap B) \neq P(A)P(B)$

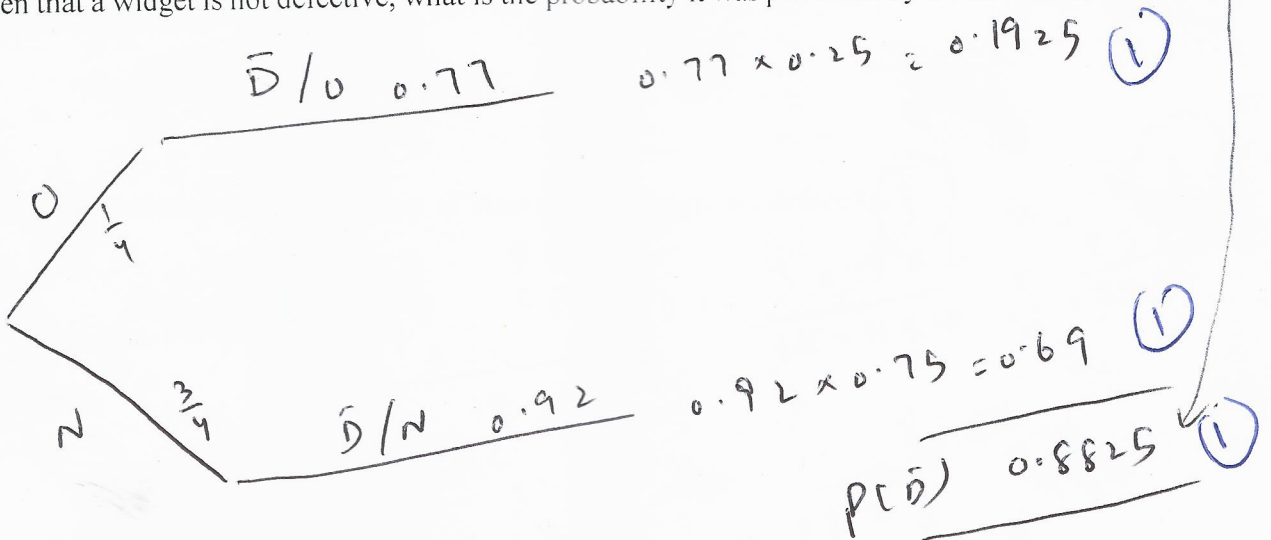
$$0.3 \neq 0.6 \times 0.4 (= 0.24)$$

Q.No.3 (2+5=7 points):- A company has 2 machines that produce widgets. An older machine produces 23% defective widgets, while the new machine produces only 8% defective widgets. In addition, the new machine produces 3 times as many widgets as the older machine does.

a. Given that a widget was produced by the new machine, what is the probability it is not defective?



b. Given that a widget is not defective, what is the probability it was produced by the new machine?



$$P(N/\bar{D}) = \frac{P(N \cap \bar{D})}{P(\bar{D})}$$

$$= \frac{0.69}{0.8825}$$

$$= 0.78187$$

Q.No.4 (3+3=6 points):- A day's production of 12 manufactured parts contains 3 parts that do not meet customer requirements. Three parts are selected randomly without replacement from the batch.

- a. Find the probability that the first part is not defective and the 2nd and 3rd are defective.

$$P(\bar{D} \cap D \cap D) = \frac{9}{12} \times \frac{3}{11} \times \frac{2}{10}$$

(1)
(1)
(1)

D	\bar{D}	T
3	9	12

- b. Find the probability that any two (out of three selected) parts are defective. (1)

$$P(2 \text{ defectives}) = \frac{\binom{3}{2} \binom{9}{1}}{\binom{12}{3}} = \frac{3 \times 9 \times 6}{12 \times 11 \times 10} = \frac{27}{220}$$

(1)
(1)
(1)

= 0.1228

Q.No.5 (3+3=6 points):- The probability that a patient recovers from a delicate heart operation is 0.8. For the next three patients who have this operation:

- a. What is the probability that exactly 2 patients survive?

$$n = 3, \quad p = 0.8$$

X : Number of patients that recover from operation
 $X \sim \text{Bin}(3, 0.8)$

$$\begin{aligned} P(X=2) &= \binom{3}{2} 0.8^2 0.2 && \textcircled{1} \\ &= 3 (0.64) (0.2) \\ &= 0.384 && \textcircled{1} \end{aligned}$$

- b. What is the average number of survived patients?

$$\mu = n p = 3 (0.8) = 2.4$$

$\textcircled{1} \quad \textcircled{1} \quad \textcircled{1}$